

JMP: A New Keynesian Model Useful for Policy

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Abstract

I add price-dispersion to a benchmark zero-inflation steady-state New Keynesian model. I do so by assuming the economy has experienced a history of shocks, which have caused the Central Bank to miss its target for inflation and output, as opposed to the conventional practice of linearizing around a non-stochastic steady state. I then allow the inflation targeting Central Bank to optimize policy. The results are truly starting.

The model simultaneously embeds endogenous inflation and interest rate persistence in an institutionally-consistent optimizing framework. This creates a meaningful trade-off between inflation and output-gap stabilization following demand and technology shocks. This resolves the so-called 'Divine Coincidence', explains the preference for 'coarse-tuning' over 'fine-tuning' and the focus in policy circles on inflation forecast targeting. When estimated the model performs well against a battery of demanding econometric tests.

Along the way, a novel econometric test of the 'Divine Coincidence' is developed- it is rejected in favor of a substantial trade-off. A welfare equivalence is derived between a class of New Keynesian models and their flexible price counterparts suggesting previous proposed resolutions may be inadequate. Finally, a novel paradox relating the 'Divine Coincidence' to 'fine-tuning' stabilization policy is derived.

1 Introduction

The macroeconomic profession has broadly settled on a *New Neo-Classical Synthesis* Goodfriend and King [1997]. This approach adds staggered price adjustment to the skeleton of the Real Business Cycle model to generate micro-founded compliments to the aggregate demand and aggregate supply equations of Old Keynesian economics. The Euler equation specifying optimal consumption and the optimal price-setting relation the New Keynesian Phillips curve (NKPC) simplify to their Old Keynesian counterparts when non-contemporaneous exogenous variables are held fixed. When this constraint is relaxed, it is necessary to specify an interest rate rule consistent with the 'Taylor Principle' to generate stable solution paths.¹ Thus a three equation set up emerges. The combination

¹The 'Taylor Principle' states that the real interest rate should be expected to increase in response to higher inflation in order to drive it back to target following a shock to expectations to prevent sunspot equilibria see Woodford [2001]. The term was coined in respect of the seminal work on monetary policy rules Taylor [1993a]. Simultaneously, Henderson and McKibbin [1993] proposed a similar rule.

of rational expectations with monetary non-neutrality allows one to analyze systematic monetary policy seemingly unhindered by the Lucas Critique Lucas Jr [1976]. For this reason, the three equation New Keynesian framework underpins the field of Dynamic Stochastic Equilibrium (DSGE) modeling, now popular in academia and with Central Banks.

Nevertheless, New Keynesianism has had problems distinguishing itself from the Neo-Classical tradition in terms of policy prescriptions and model predictions. Macroeconomics as a separate intellectual discipline came into existence following *The General Theory* Keynes [1936] in order to learn how best to mitigate inefficient business cycle fluctuations. The lack of inefficient fluctuations in benchmark Real Business Cycle models, such as Long Jr and Plosser [1983] and Barro and King [1984], is the most unappealing aspect of Neo-Classical modeling and the reason why it has never enjoyed favor in policy circles.

Unfortunately, comparable New Keynesian models also suffer from this problem. Correia et al. [2008] show that the Central Bank can implement the social optimum when the government is using a standard set of distortionary tax instruments to correct static market failures. Woodford [2000] Michael [2002] show an optimal monetary policy can successfully stabilize inflation and the output gap simultaneously under a wide variety of shock processes. This result has been labeled the 'Divine Coincidence' Blanchard and Galí [2007] (henceforth DC).² It is an anathema in policy circles. 'Inflation nutters' is the uncharitable description former Bank of England Governor Mervyn King gave to those advocating complete inflation stabilization as a policy objective, as if the DEC applied King [1997].

This reflects a fundamental disjuncture between optimal monetary policy in theory and successful policy practice. I show that in the stochastic complement to the DC framework deviations of inflation and the output gap from target should be white noise. Therefore in the limit as Central Banks become better able to observe shocks in real time and change policy rates more frequently the Central Bank can 'fine-tune'³ away all fluctuations in inflation and the output gap.

Secondly, there is an 'inflation persistence puzzle'. In the data inflation appears persistent across all nations, time periods, policy regimes, levels of aggregation and plausible assumptions about trends in other macroeconomic variables O'Reilly and Whelan [2005] Pivetta and Reis [2007] Gerlach and Tillmann [2012] Imbs et al. [2011] Meller and Nautz [2012] Kouretas and Wohar [2012] Plakandaras et al. [2014] Vaona and Ascari [2012] Tillmann [2013] Choi and O'Sullivan [2013] Nakamura and Steinsson [2013].⁴ The forward-looking NKPC

²The possibility of a binding zero bound on nominal interest rate overturns this conclusion, in particular it is optimal for the Central Bank to allow a period of above target inflation and output immediately following a zero bound spell Eggertsson et al. [2003]. However, the problem may reoccur if we adopt the more empirically credible assumption that Central Banks can mimic negative nominal interest rates through quantitative easing.

³The term is frequently attributed to Walter Heller Chief Economic Adviser to President Kennedy see for example <http://connection.ebscohost.com/c/reference-entries/40422478/fine-tuning-1960s-economics>. It referred originally to fiscal policy in an 'Old Keynesian' set up. Scepticism about the concept was focal to monetarist opposition to traditional Keynesian macroeconomics Snowdon and Vane [2005] see for example Friedman [1968].

⁴It is worth noting that many studies are able to reject the null of no persistence in inflation even when they have sufficient power to uncover statistically significant changes in persistence across policy regimes. Although, there is considerable heterogeneity in inflation persistence across sectors macroeconomic persistence is not a figment of aggregation bias.

is strongly rejected in favor of a hybrid specification containing lagged as well as future inflation, a result that has not been adequately explained in a consistent theoretical fashion see Roberts [1997]Galí and Gertler [1999] Rudd and Whelan [2005]Fuhrer [2006]Whelan [2007]Rumler [2007]amongst a voluminous literature. In the DC model inflation has no persistence. Worse still when DC is relaxed by allowing persistent distortionary shocks⁵, optimal policy amounts to a form of price-level targetingWoodford [2010]. Therefore, inflation inherits *negative persistence*.

This contrasts with best practice among inflation targeting Central Banks who practice so called 'coarse-tuning'Lindbeck [1992]. They realize that inflation possess intrinsic persistent so that they cannot hit inflation and output targets in every period. Instead they practice so called *inflation forecast targeting*KohnSvensson [2010]Svensson [2012]. This is where policy and projections for future policy are adjusted to yield a desirable expected path for inflation and real activity consistent with *medium term* stability. Usually defined as forecast inflation and output gap sufficiently close to target after a time frame of 18 months to 3 years⁶.

The three equation framework was designed explicitly to address issues with optimal monetary policy and its effects upon inflation and real activity. New Keynesian theory is failing the test of policy relevance.Chari et al. [2009] were right New Keynesian models are not yet fit for purpose.

A second challenge confronting New Keynesian modeling is the policy persistence puzzle. Interest rates are highly persistent- much more so than underlying shock processes. This means that estimated Taylor rules require coefficients on lagged rates near unity to purge serial correlation and represent an optimizing relationshipCoibion and Gorodnichenko [2012]Vázquez et al. [2013]⁷ Attempts to explain why this might be optimal have so far proven unconvincing. It is widely suspected, the interest rate and inflation persistence puzzles are related. Several attempts at a simultaneous resolution have been made.Cogley and Sbordone [2008] and Cogley et al. [2010] have attempted a simultaneous resolution by incorporating shocks to the trend rate of inflation. My analysis most closely

⁵Distortionary shocks effect the wedge between actual and efficient output. They enter into the NKPC where they are often called 'cost-push' shocks their interpretation and misinterpretation are discussed in Section 4.

⁶Studies with VARs and policymakers wisdom suggest that it takes between 18 months and two years for a change in monetary policy to have its maximum impact on inflation. This result seems to be robust across changes in policy regimes; Orłowski [2000](see p 315-320)Batini and Nelson [2001]Gerlach and Svensson [2003]. On its website the Bank of England advises the general public that: "Monetary policy operates with a time lag of about two years." <http://www.bankofengland.co.uk/monetarypolicy/Pages/overview.aspx> However the Bank publishes forecasts three years ahead and frequently talks about "inflation returning to target by the three year horizon" consistent with a longer view of the stabilization and empirical work by Havranek and Rusnak [2013] period.<http://www.bankofengland.co.uk/publications/Pages/inflationreport/infrep.aspx> Practices are similar at other leading inflation targeting Central Banks.

⁷The papers control respectively for non-rational expectations on the part of the policymaker and data revisions. See alsoRudebusch [2002]Petra [2004]Rudebusch [2006]Carrillo et al. [2007]Conraria et al. [2014]. The acclaimed Norges Bank (the Central Bank of Norway) chooses to insert a substantial interest rate stabilization term- ad hoc with respect to its policy mandate- into its loss function used to derive optimal policy- in order to generate policy rate predictions consistent with credible application of inflation forecast targetingBergo [2007]Holmsen et al. [2007]. Other Central Banks swerve around this problem by using subjective judgments or information from futures rates or the yield curve which have expectations of policy persistence built-in Ang et al. [2007] Hamilton et al. [2011].

follows Alves [2014] who uses trend inflation to generate inflation persistence and resolve the DC. I show my resolution is not only more parsimonious but more empirically credible than adding shocks to trend inflation.

I am able to derive a resolution to the inflation, persistence and optimal policy paradoxes simultaneously. I show that trend inflation is not necessary to resolve the DC all that is required is to appreciate that a history of shocks to the economy will generate price dispersion and therefore the New Keynesian model should be linearized around a stochastic steady state with price dispersion rather than the non-stochastic case with zero price dispersion as is currently ubiquitous⁸ This allows New Keynesianism to fully differentiate its welfare implications from those of Real Business cycle- verifying the suspicion long held by policymakers and amongst academics that provided there is some nominal rigidity inefficient business cycle fluctuations will take place, that cannot be fully mitigated by Central Bankers, such that substantive trade-offs exist in monetary policymaking. Price dispersion is the internal combustion engine driving the New Keynesian car.

I discuss advantages of my resolutions to these paradoxes over the trend inflation approach and others in the literature. The result is what I believe to be a New Keynesian model fit for purpose. The paper proceeds as follows section 2 derives the simplest New Keynesian model focusing on its non-stochastic zero inflation steady state. Section 3 considers optimal policy and formalizes the Divine Coincidence (DC). This includes a cross-country econometric investigation. Section 4 considers optimal policy and persistence problems. Section 5 is the major contribution of the paper. It is where I derive optimal policy under inflation targeting with price dispersion. Section 6 covers simulation and estimation of the complete model. Section 7 augments the basic model with nominal wage rigidity. Section 8 derives paradoxes related to 'fine-tuning' and welfare equivalence for a class of New Keynesian models with real distortions. Finally, Section 9 draws conclusions and suggests directions for future research.

2 New Keynesian Model

This section exposit the benchmark New Keynesian model which forms the basis for subsequent analysis with emphasis on aspects pertinent for forthcoming results.

2.1 Household's Problem

There is a representative household which solves the following problem:

$$\max_{C_t, l_t} E_t \sum_{T=t}^{\infty} \beta^{T-t} [u(C_T) - \varphi_T \nu(l_T)] \psi_T \quad (1)$$

subject to the Budget Constraint:

$$P_t C_t + B_t = (1 + i_{t-1}) B_t + P_t W_t l_t + \int_i \Pi_t(i) di \quad (2)$$

⁸This approach is similar to that of Juillard and Kamenik [2005] Michel [2011] coeurdacier2011risky although their focus on deriving risk premium causes them to consider perturbations of order two and higher which I do not need.

β is the discount factor, C refers to aggregate consumption whilst l is labor supply. B refers to the holding of one period risk free nominal bonds. i_t is the risk-free nominal interest rate paid at the end of period t on the bond. P is the price level- bonds are the numeraire here. W is the real wage. Finally Π_i is profit from an individual firm i .

Note in a stochastic environment firms need not make the same profits even with a symmetric equilibrium. Since when price rigidity is introduced firms with the same demand curve with the same demand curve will charge different prices depending on when they last reoptimized and can therefore make different levels of profit. ψ_T and φ_T are disturbance terms. The former affects intertemporal consumption preferences the latter the willingness to supply labor. In section 3 more sophisticated interpretations for these stochastic terms will be discussed. The budget constraint states that the uses for nominal income consumption and saving must be equal to the sources of income wealth, labor and dividend income.

Consumption is desirable but working is undesirable so u and ν are increasing. u is concave to incentivize consumption smoothing whilst ν is convex to encourage workers to take leisure. An Inada condition on u , a restriction on the process governing ψ , a zero initial wealth condition and a transversality condition serve to ensure an interior solution. They are as follows:

$$\lim_{c \rightarrow 0^+} u'(c) = \infty \quad (3)$$

$$B_0 = 0 \quad (4)$$

$$E_t \int_S \sum_{T=t}^{\infty} \beta^{T-t} [u(C_T) - \varphi_T \nu(l_T)] \psi_T < \infty \quad (5)$$

for *a.e.* $S \in S'$ Where S is any member of S' the σ -algebra generated by the stochastic process F , which assigns probability measure to the individual shock processes ψ and ϕ although we could imagine the model being augmented with further disturbance processes, reflecting additional sources of macroeconomic variability.

$$\lim_{T \rightarrow \infty} E_t \frac{B_T}{P_T} u_C(C_T) \geq 0 \quad (6)$$

Equation (3) is a "no-starvation" condition, it ensures the agent will always choose to consume even though working is costly. Equation (4) stops the agent living off their savings and allows me to avoid articulating a specific government budget constraint. Together (3) and (4) ensure an interior labor supply. Equation (5) ensures local uniqueness by stopping the objective function exploding. Finally, equation (6) is a "No-Ponzi" condition it forces the agent to honor the present value of their debts. Due to monotonicity of the utility function the constraint will always bind with equality and if it were left out the agent would simply borrow an infinite amount and never repay.

The first order conditions for the households are as follows:

$$u_c(C_t) = (1 + i_t) \beta E_t U_c(C_{t+1}) \frac{\psi_{t+1}}{\psi_t} \frac{P_t}{P_{t+1}} \quad (7)$$

$$u_c(C_t) W_t = \phi_t \nu_l(l_t) \quad (8)$$

Equation (7) the so-called Euler equation specifies the path for optimal consumption, whilst equation (8) which equates the marginal costs and benefits of working, yields the labor supply curve. When the market is approximated the following parameters of the utility function will be used $\sigma = \frac{-C_{u_{cc}}}{u_c}$ and $\eta = \frac{l_{\nu l}}{\nu_l}$ both are strictly positive to ensure an interior solution. They measure respectively the concavity of consumption utility and the convexity of the disutility from work. σ is the inverse of the inter-temporal elasticity of substitution- the consumer's willingness to shift consumption across time periods, similarly η is inversely related inter-temporal elasticity of labor supply.⁹

I now specify that the measure of the firms is a unit continuum. The firms are monopolistically competitive. This ensures that firms face a meaningful pricing decision, avoiding the case of unbounded sales possible under perfect or simple Bertrand competition. Aggregate consumption by the household can now be described as an constant elasticity aggregator over the consumption of each variety of good¹⁰ $C_t = [\int_0^1 c_t(i)^{1-\theta} di]^{\frac{1}{1-\theta}}$ where $\theta > 1$ is the elasticity of substitution among varieties. The optimal consumption allocation across varieties yields the demand system $c_t(i) = (\frac{p_t(i)}{P_t})^{-\theta} C_t$. Combining the two yields the expression for the aggregate price level in terms of all the prices in the economy $P_t = [\int_0^1 p_t(i)^{1-\theta} di]^{\frac{1}{1-\theta}}$. Note that all production is consumed so $c_t(i) = y_t(i)$ and

$$C_t = Y_t \tag{9}$$

where the subscript i refers to an individual firm Y_t refers to aggregate output.

To introduce nominal rigidity I adopt the Calvo model Calvo [1983] Yun [1996] as it is the most popular in the literature. Although, major theoretical results about the behavior of price dispersion and welfare implications generalize to alternative models of nominal rigidity- a point demonstrated in Section 3 and Appendix A. In the Calvo model when a firm selects a price it stays constant for a stochastic number of periods, the probability that the firm is offered an opportunity to change its price is the same each period and across firms. It is equal to $1 - \alpha$, hence α the fraction of prices that automatically stay fixed in a given period is a measure of the degree of nominal rigidity. Firms are assumed to maximize profits. Their optimal pricing problems are symmetric and I focus exclusively on symmetric equilibrium. Therefore, all firms who reset their price in any given period will select the same price. The measure used to compute the aggregate price and consumption will be discrete with mass points at every previous optimal reset price. Each firms price will be the optimal price when they were last able to reset it. The current reset price will be denoted p_t^* .¹¹

⁹ σ is also the coefficient of relative risk aversion but this interpretation is not relevant here because I only consider first order approximations where certainty equivalence holds.

¹⁰See ARMINGTON [1969] for an early application this functional form was popularized by Dixit and Stiglitz [1977] and is sometimes called after them.

¹¹There are several alternative approaches to building nominal rigidity into the pricing problem. The next most similar is Taylor contracting model see Taylor [1979] where firms change their price every $\frac{1}{1-\alpha}$ periods. The model has the same log-linear form. State-Dependent pricing models where firms have the opportunity for a costly price-change every period, at least in their benchmark forms struggle to generate aggregate nominal rigidity Caplin and Spulber [1987] Golosov et al. [2007] Head et al. [2012]. Price and wage rigidity have been extensively documented across countries with different structural characteristics and policy environments Dhyne et al. [2006] Dickens et al. [2007] Babecky et al. [2010]. There has been considerable success in modeling how the heterogeneous pricing behavior of individual firms might give rise to aggregate monetary non-neutrality Guimaraes and Sheedy [2011] Dixon

2.2 Firms' Problem

Firms maximize profits in the choice of factors and also in prices when they are given the chance to reoptimize. In order to profit maximize they must cost minimize.

2.2.1 Cost Minimization

I make two simplifications here first that labor is the only factor of production, second that there are constant returns to scale. Appendix E ??? relax these assumptions, in fact allowing for decreasing returns to labor actually strengthens the main mechanism in this paper. The problem is therefore as follows:

$$\min_{l_t(i)} W_t l_t(i) \quad (10)$$

subject to

$$c_t(i) = A_t l_t(i) \quad (11)$$

Hence we can solve for real marginal cost

$$\varphi_t = \frac{W_t}{A_t} \quad (12)$$

φ the real marginal cost is equal to the ratio of the real wage to aggregate technical efficiency term A_t , from the RBC model. More detailed interpretations will be offered in Section 4. Firm level productivity shocks either do not exist or have been averaged out by a law of large numbers. Note that firms will have identical marginal costs here. This is because they face the same input prices and there are constant returns to scale. It simplifies my analysis but both assumptions are relaxed in Appendix items to allow for capital formation and nominal wage rigidity.

2.2.2 Profit Maximization

The firm maximizes the expected present value of profits as follows:

$$\max_{p_t(i)} E_t \sum_{T=t}^{\infty} (1-\alpha)^{T-t} Q_{t,T} \left[\frac{p_t(i)}{P_T} y_T - \mathfrak{I}_T y_T \right] \quad (13)$$

s.t. demand and market clearing constraints:

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} C_t \quad (14)$$

Here $Q_{t,t+k} = \beta^k \frac{U_C(C_{t+k})}{U_C(C_t)}$ represents the real stochastic discount factor (SDF), it is the risk-adjusted present value of future consumption k periods ahead. Optimal pricing gives the rest price p_t^* as a markup $\frac{\theta}{\theta-1}$ over expected future marginal costs. The markup reflects the degree of imperfect competition which is declining in θ the degree of substitution among varieties. The limiting case $\theta \rightarrow \infty$ returns perfect competition. The problem is infinite horizon, the discounting

and Le Bihan [2012] Anderson et al. [2013] Kehoe and Midrigan [2014]. I have abstracted from wage rigidity initially for simplicity because the labor market is not my primary focus.

reflects both Q the real SDF and α governing the life-expectancy of the price. In real terms we have:

$$\sum_{T=t}^{\infty} \Phi_T \left[\frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} \mathfrak{P}_T \right] = 0 \quad (15)$$

Where the discount weight $\Phi_T = Q_T y_T$ reflects the SDF and the scale of output at time T produced by the firm given that it had last set its price in period T . Under Calvo pricing the price level evolves as follows:

$$P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)(p_t^*)^{1-\theta} \quad (16)$$

The persistence of the price level depends on α the degree of rigidity.

2.3 Policy Rule

Finally, it is necessary to have a policy rule to close the model. In practice none of the models used in empirical simulations can be viewed as optimizing a suitable loss function. The most well-known is the so-called Taylor rule¹²¹³ popularly formulated as follows:

$$i_t = i_t^* + a_\pi \tilde{\pi}_t + a_y \tilde{y}_t \quad (17)$$

¹²The alternative is to specify a money supply rule. The most popular money supply rule was the McCallum rule McCallum [1988]. The McCallum rule is essentially a flexible version of the monetary targeting rules advocated by monetarist like Milton Friedman Friedman and applied by Western policymakers as part of disinflationary efforts during the late 1970s and early 1980s. In Britain and the United States monetary targeting was eventually abandoned owing to perceived instability in the money demand function impaired implementation. It now seems that this appearance was a figment of a miss-measuring the opportunity cost associated with the new money substitutes on offer as a consequence the financial deregulation occurring around that time- Ericsson et al. [1998] Ireland [2009] Barnett [2012] Ball [2012] Lucas and Nicolini [2015]. In the stylized models expositied here money demand is inconsequential to policy because of the presence of frictionless financial markets as shown by Woodford [1998]. The empirical application of the result is doubtful however, as it appears empirical specifications including money perform better than those with just interest rates Belongia and Ireland [2014]. This seems to be particularly important in periods of very low interest rates- where the interest rate channel breaks down but policy still appears be effective Ueda [2012] Kapetanios et al. [2012] D'Amico et al. [2012] Swanson and Williams [2014] Gilchrist et al. [2015]. The problem appears to be the restriction on the class of financial market inefficiency imposed in benchmark models. It has been suggested that these distortions may have substantial effect on monetary policy propagation in normal times Jimémez et al. [2014] Gertler and Karadi [2015] Nelson et al. [2015] and optimal policy Chadha et al. [2014] Ellison and Tischbirek [2014] de Groot [2014]. Nevertheless, results tend to be model specific and may dependent on other aspects of the policy and regulatory environment Svensson [2014]. Furthermore, macro-prudential concerns does not seem to have played a major role in monetary policy determination during the estimation period considered in Sections 2 and 6 Fuhrrer and Tootell [2008]. Fostering financial stability does not appear to have been part of the monetary policy frameworks in any nations considered in Section 4 Rotemberg [2014]. There are several example of leading policymakers argue Central Banks should not try to burst bubbles Bernanke and Gertler [2001] Posen [2006]. These concerns support my decision to work with a benchmark efficient financial markets setup. The evidence about Quantitative Easing is the main reason why I exclude the post-crisis period.

¹³It is worth noting that even though they are associated with inflation targeting Taylor rules seems to explain policy-making just as well even when the Central Bank professes to be following a different policy regime such as money targeting see Bernanke and Mihov [1997] Clarida et al. [1998]. This increases my confidence in applying a micro-founded inflation targeting model to the United States which has never been an explicit inflation targetter; later on in the paper.

Where $a_\pi > 1$ and $a_y \geq 0$ $\tilde{\pi}$ refers to the inflation gap the difference between observed inflation π_t and its target π_t^* \tilde{y}_t is the output gap defined as the difference between the actual output y_t and its natural or potential rate \bar{y}_t . This potential rate will be proxied by either deterministic time trend or a stochastic trend derived from conventional settings of the Hodrick-Prescott filter Hodrick and Prescott [1997]. Here the potential rate is given by the flexible price equilibrium derived above. A new definition of the theoretical concept of the potential rate will be given in Section 5. i_t^* is the natural nominal rate of interest. It comprises a natural real rate r_t^* and the inflation target π_t^* .¹⁴ Essentially the rule states that nominal interest rates should rise more than one-for-one (so the real interest rate increases) with inflation to ensure a stable solution. There is also an allowance but not a requirement for interest rates to smooth the output gap¹⁵. There are no lags or leads in the relationship so contemporaneous economic developments determine current policy. This is intuitive, there is no benefit to conditioning on past variables in a forward-looking model¹⁶. In the particular but instructive case with no persistence to exogenous shock processes the future of a forward-looking system will be identical in expectation to the steady state. In a forward-looking system one instrument per period should be sufficient to implement optimal policy.

Note, we are not required to believe that interest rates are determined simultaneously with output and inflation. In subsequent sections interest rates will be determined before output and inflation are realized. As the rule is linear with rational expectations the expectation errors will pass into the white noise error term- providing in fact the main rationale for the regression error itself. I interpret the policy rule as a description of actual policy for a Central Bank with rational expectations¹⁷ and test its fit to the data. Hence, I view the presence of serial correlation and a significant lagged interest rate term as a rejection of Taylor rule as a description as a model of real-world policy-setting.

2.4 Price Dispersion

To understand the dynamics of the system we need to linearize it, for business cycle interpretation we need to choose a point around which to carry out this perturbation that could be interpreted as a long-run equilibrium. Economists have always used the *non-stochastic steady state* that would prevail if the economy were never subject to shocks or expected to be so. The non-stochastic

¹⁴For generality I allow for a time-varying inflation target. In this section and the associated Appendix A as in Section 6 I exploit this feature when testing my model on United States data. I do so because the US has never had a legal inflation target and there are suggestions that monetary policy behavior has changed over time e.g. with changes in government or Federal Reserve Chairperson. In section 4 however, when testing optimal policy in countries with a legal inflation target I take the inflation target as fixed throughout the period in the main specifications.

¹⁵There are several differences from Taylor's original rule.

¹⁶The system is 'forward-looking' in the sense that agents have rational expectations and there are no state variables.

¹⁷There is some controversy about whether Central Banks do display rational expectations see Romer and Romer [2008] and Ellison and Sargent [2012]. However, the conclusion of excess persistence appears robust to and even strengthened by the use of internal forecasts understood to be the best forecast of the evolution of the economy see Coibion and Gorodnichenko [2012]. In any case the significance and substantiveness of persistence appears too large to be explained solely by deviations from rational expectations.

steady state is equivalent to the flexible price equilibrium. The point of this paper is that this approach is misguided providing an erroneous interpretation of business cycle dynamics.

The New Keynesian framework differs from Neoclassical models by preventing every firm re-optimizing prices in every period. This allows for the possibility of *price rigidity* where today's price level contains reset prices from previous periods, as well as the current optimal reset prices. This means there can be *price dispersion* with implications for resource allocation and welfare.

We characterize price dispersion using the demand aggregator

$$\Delta = \int_i \left(\frac{p_i}{P}\right)^{-\theta} d\mu_i \quad (18)$$

It appears in the market-clearing condition

$$\Delta_t C_t = A_t L_t \quad (19)$$

Under Calvo pricing Δ evolves according to the following relationship:

$$\Delta_t = (1 - \alpha) \left(\frac{p_t^*}{P_t}\right)^{-\theta} + \alpha(1 + \pi)^\theta \Delta_{t-1} \quad (20)$$

Using equation (16) to eliminate the reset price we find:

$$\Delta_t = (1 + \pi)^\theta \frac{(1 + \pi - \alpha)^{\frac{\theta}{\theta-1}}}{(1 - \alpha)^{\frac{1}{\theta-1}}} + \alpha(1 + \pi)^\theta \Delta_{t-1} \quad (21)$$

Price dispersion Δ is a strictly convex function of inflation π and is a persistent process with the degree of persistence increasing in the degree of price rigidity α . These two features underpin the rest of the analysis in this paper. To understand the dynamics it is necessary to log-linearize the system. The following property is remarkable.

Lemma 1. *Around the non-stochastic steady state $(\pi_t, \Delta_t) = (0, 1)$ the log-linear approximation $\hat{\Delta}_t = 0$ for all values of inflation π_t .*

Proof. Simply carry out log-linearization of the the system (Π, Δ) where $\Pi_t = \frac{P_t}{P_{t-1}}$ and used the fact that $\hat{\Pi}_t = \pi_t$ to obtain:

$$\hat{\Delta}_t = \varkappa(\pi, \Delta) \pi_t + \alpha(1 + \pi)^\theta \hat{\Delta}_t \quad (22)$$

where (π, Δ) are the values about which the approximation is being taken. $\varkappa(\pi, \Delta)$ can be expressed as follows:

$$\varkappa(\pi, \Delta) = \frac{\theta(1 + \pi)^{\theta-1} \pi}{\Delta(\theta - 1)(1 - \alpha)^{\theta-1}} \varpi(\pi, \Delta)$$

$$\varpi(\pi, \Delta) = (1 + \pi - \alpha)^{\frac{\theta}{\theta-1}} (\theta - 1) + (1 + \pi)(1 + \pi - \alpha)^{\frac{1}{\theta-1}} + \alpha(\theta - 1)(1 - \alpha)^{\theta-1} \Delta$$

$\varkappa(\pi, \Delta)$ is strictly positive except crucially where $\pi = 0$ and the inflation term drops out as $\varpi(0, \Delta) = 0$. This follows from the fact that $1 + \pi - \alpha > 0$, is the requirement for the non-negativity of the reset price p^* from price level construction equation (16). Intuitively, in a New Keynesian model the prices

that stay fixed put a lower bound on the admissible rate of inflation¹⁸. By assumption of a non-stochastic steady state $\hat{\Delta}_{t-1} = 0$ hence as the sum of two zero terms $\hat{\Delta}_t = 0$ regardless of the value of inflation π_t . \square

2.4.1 Flexible Price Equilibrium

If all firms can adjust prices in every period corresponding to the limiting case $\alpha = 0$ (15) reduces to:

$$\frac{p_t^*}{P_t} = \frac{\theta}{\theta - 1} \varphi_t = \bar{\mu} \varphi_t \quad (23)$$

Where $\bar{\mu}$ is the markup. With flexible prices all firms charge the same price so $p_t^* = P_t$ hence we can solve for the marginal cost which is inversely proportional to the markup. As the flexible price equilibrium has to be interpreted as a non-stochastic steady-state of the New Keynesian model we must set the disturbance term to its equilibrium value so that $\bar{\mu}_t = \bar{\mu} = \frac{1}{\mu}$. Likewise, with $\Delta = 1$ the market-clearing relation (19) simplifies to $C_t = \bar{A}L_t$ combining with the labor supply relation it is possible solve for the steady state. A natural rate of interest \bar{r} set by the Euler equation brings about equilibrium over time. I am not going to do so here, as the non-stochastic steady state is a special case of the stochastic steady state which I shall characterize in Section 6. The only source of inefficiency in this steady-state comes about from the markup. This inefficiency can easily be corrected by an appropriate tax and subsidy scheme¹⁹ to leave a Pareto efficient allocation I have left it out for comparability with my model where I prove that Pareto efficiency is not implementable and use an alternative welfare criterion to characterize optimal policy.

2.5 New Keynesian Phillips Curve and the Forward-Solution

Analysis of the NK model begins with the recursive marginal cost Phillips curve. It describes how current inflation is determined by the present deviation of real marginal costs from steady state and the expectation of next periods inflation. It is derived from combining the optimal price-setting condition (15) and the price level construction equation (16) and log-linearizing²⁰.

$$\pi_t = \kappa \hat{\mu}_t + E_t \beta \pi_{t+1} \quad (24)$$

¹⁸This result can be modified to models with nominal indexation schemes considered in the literature with the minimum bound being a function of the indexing variable π_{t-1} or trend inflation $\bar{\pi}$. This will not come into play in my set-up because the trend rate of inflation will be set to zero for congruity with the benchmark model and to allow me to fully overturn the 'Divine Coincidence'. Alternatively π_t can be viewed as the detrended rate of inflation with each firm indexing to trend in every period. There is very little empirical support for nominal price indexation. However, I will leave relaxation of this assumption to future research.

¹⁹The most plausible such scheme would be a sales tax rebated lump sum to households. The results in the next section imply that any tax scheme to mitigate the two inefficiencies from price dispersion and imperfect competition would have to vary over time with the level of price dispersion. This does not seem a good characterization of real world fiscal policy and would add unnecessary complexity to my model.

²⁰Consult Walsh [2010] for step-by-step derivations of this and other aspects of the basic New Keynesian model.

Where $\kappa = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}$ the corresponding infinite horizon forward solution for inflation in terms of future marginal costs is

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t \hat{\mu}_{t+i} \quad (25)$$

It is convenient to eliminate the unobserved variable $\hat{\mu}_t$ representing real marginal costs with an expression for the output gap which can in principal be estimated from macroeconomic data. First linearize the marginal cost function (12)

$$\hat{\mu}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t \quad (26)$$

Next linearize the production function

$$\hat{a}_t = \hat{y}_t - \hat{l}_t + \hat{\Delta}_t \quad (27)$$

With the assumption of a non-stochastic steady state we know that $\hat{\Delta}_t = 0$ now using optimal labour supply condition (8) we find

$$\hat{\mu}_t = (\sigma + \eta) \left[\hat{y}_t - \frac{1 + \eta}{\sigma + \eta} \hat{a}_t \right] \quad (28)$$

Now the current New Keynesian model uses the *efficient output gap* y^e defined as the log-difference between actual output \hat{y}_t and the flexible price equilibrium output \hat{y}_t^f

$$y_t^e = \hat{y}_t - \hat{y}_t^f \quad (29)$$

Note that flexible price equilibrium output is $\hat{y}_t^f = \frac{1+\nu}{\sigma+\nu} \hat{a}_t$ so the productivity term \hat{a}_t cancels out of the marginal cost expression which is proportional to the output gap

$$\hat{\mu}_t = (\sigma + \eta) y_t^e \quad (30)$$

This yields the conventional recursive Phillips curve where inflation is a function of the efficient output gap and next periods expected inflation

$$\pi_t = \omega y_t^e + \beta E_t \pi_{t+1} \quad (31)$$

Where $\omega = (\sigma + \eta) \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}$. Its forward solution is

$$\pi_t = \omega \sum_{i=0}^{\infty} \beta^i E_t y_{t+i}^e \quad (32)$$

Accepting that we cannot have a perfect fit to test the model we need error terms in each equation therefore the final system are the following Euler, Taylor and Phillips curve triplet. In this section I neglect the possibility of shocks to the natural rate $\bar{r}_t = \bar{r}$ this is picked up in Section 5 where it is shown to be inconsequential for the behavior of inflation or the efficient output gap.

$$y_t^e = E_t y_{t+1}^e - \frac{1}{\sigma} (i_t - \bar{r} - E_t \pi_{t+1}) + u_t^1 \quad (33)$$

$$i_t = \bar{r} + a_\pi \pi_t + a_y y_t^e + u_t^2 \quad (34)$$

$$\pi_t = \omega y_t^e + \beta E_t \pi_{t+1} + u_t^3 \quad (35)$$

To make the calculation of the forward solution easier I follow the convention of eliminating the policy rule to give the matrix system:

$$X_{t+1} = \begin{bmatrix} \pi_{t+1} \\ y_{t+1}^e \end{bmatrix} = AX_t + Bu_t \quad (36)$$

Where $X_t = \begin{bmatrix} \pi_t \\ y_t^e \end{bmatrix}$ $A = \begin{bmatrix} \beta^{-1} & -\omega\beta^{-1} \\ \sigma^{-1}(a_\pi - \beta^{-1}) & 1 + \sigma^{-1}(a_y + \omega\beta^{-1}) \end{bmatrix}$

$$B = \begin{bmatrix} 0 & 0 & 1 \\ -1 & \sigma^{-1} & \sigma^{-1}\beta^{-1} \end{bmatrix} (u_t)' = [u_t^1 \quad u_t^2 \quad u_t^3]$$

From the general solution

$$X_t = \sum_{i=0}^{\infty} -A^{-(1+i)} Bu_{t+i} \quad (37)$$

The three variables can be expressed as sums of the expected future shock terms.

$$\pi_t = \sum_{k=0}^{\infty} \zeta_\pi^1 u_{t+k}^1 + \zeta_\pi^2 u_{t+k}^2 + \zeta_\pi^3 u_{t+k}^3 \quad (38)$$

$$y_t = \sum_{k=0}^{\infty} \zeta_y^1 u_{t+k}^1 + \zeta_y^2 u_{t+k}^2 + \zeta_y^3 u_{t+k}^3 \quad (39)$$

$$i_t = \bar{r} + \sum_{k=0}^{\infty} \zeta_i^1 u_{t+k}^1 + \zeta_i^2 u_{t+k}^2 + \zeta_i^3 u_{t+k}^3 \quad (40)$$

The details of the ζ coefficients are not important here and are reported in Appendix A. The Blanchard-Kahn condition Blanchard and Kahn [1980] that both eigenvalues of the matrix A lie outside the unit circle is required for the series to converge to a unique solution.

2.6 Persistence Problem

The persistence problem of the New Keynesian model lies in the property of the errors and expectations. To link the two it is assume imperfect information.

Assumption 1. *The central bank has imperfect information about the present state of the economy- which is resolved at the end of the period after they have chosen their behavior.*

This restriction seems realistic for example quarterly output figures are released soon after the end of the relevant quarter,so the central bank only observes present state of the economy with error this allows us to interpret the error term in the Taylor rule u_t^2 as a central bank expectation error.²¹

Proposition 1. *Each error term u_t^i must be white noise i.e. $Eu_t^i | \mathcal{I}_{t-l} = 0, \forall t > 0, l \leq t$*

²¹In reality private sector agents should be equally if not more uncertain about the state of the economy. However, we do not need to assume imperfect information to interpret error terms u_t^1 and u_t^3 as expectation errors since these equations already contain the unknown variable $E_t \pi_{t+1}$. Indeed introducing imperfect information to the private sector would require the unnecessary complication of a signal extraction problem.

Here \mathcal{I}_T is the information set provided by the model at time T or else the model is observationally equivalent to a model with bounded rationality (irrational expectations) and is therefore not identified in the encompassing class of DSGE models. This result follows from the fact that when we estimate the model with macroeconomic data we cannot observe expectations. Therefore we must use the mathematical expectation given by the structural model denoted by a superscript M . Take the example of the Phillips curve for concreteness $E_t \pi_{t+1} = E_t^M \pi_{t+1}$. Assumption 1 makes this easily applicable to both other equations in the system. Using S to denote 'subjective' and assuming there are no other I can subsume the Phillips curve error term u_t^3 into the subjective expectation yields $E_t^S \pi_{t+1} = E_t^M \pi_{t+1} + u_t^3$. Now if u_t^3 is not white noise then the agent is making systematically incorrect predictions and we have observational equivalence with a bounded rationality (irrational expectations) model. Therefore the rational expectations model is not identified²².

For this reason apart from the technology shock, that as shown above does not feature in the final solution of the benchmark New Keynesian model, when I estimate my own model in Section 7 I keep all other shocks white noise for congruity with the rational expectations framework. It is retained however, in Sections 5 and 6 where optimal policy results are derived and tested. In Section 5 more sophisticated economic interpretations of shocks in particular u_t^3 will be discussed.

Assumption 2. u_t^i is an iid random variable with $E|u_t| < \infty$

This restriction will be used both to derive the conditions for existence of a solution to (33)-(35) and construct arguments about the consistency properties of various estimators. At the cost of additional complexity weaker conditions could be used to accommodate dependence of higher moments. In the empirical section, processes with higher order dependence such as GARCH will be considered- all of which will admit solutions for the variables (y_t^e, i_t, π_t) .

Note that I have not ruled out time trends or unit roots in actual output y_t - provided that any non-stationary process impacts actual output y_t and flexible price output y_t^f equally then the efficient output gap y_t^e could be stationary. In Section 3/(Appendix?) a battery of tests reject a unit root in inflation in both panel and time series applications. In Section 6/(Appendix?) reports similar findings for interest rates.

In empirical application (section 6 or 7) the output gap variable will automatically be stationary because of the detrending procedure. It is worth noting that time series of output distributions in OECD countries are well-described by the exponential power family of distributions- for which all power moments are defined- see Christiano [2007] Fagiolo et al. [2007] Fagiolo et al. [2008] Fagiolo et al. [2009] Franke [2015]. It is natural to expect that Keynesian stabilization policy would make the tails of the output gap distribution thinner than that of raw output²³ Indeed I am able to confirm this intuition in Section 5/6 when

²²An alternative strategy would be to relax rational expectations and use survey data to measure inflation expectations Roberts [1995] Roberts [1997] Coibion [2010], although I do not pursue this here

²³In fact Fagiolo et al. [2009] shows that the exponential power family distribution is able to encompass output fluctuations across different filtering specifications- including unfiltered. Several cited studies bolster their case with similar findings with similar findings for other macroeconomic time series such as real wage and employment series that would inherit their

I derive my own Keynesian model and characterize optimal policy. When I simulate and test my own model in Section 6/7 I use a normal distribution for comparability with existing research. Nevertheless, persistence results in Section 3 are robust to error distributions in this class. Imposing these two assumptions allows me to characterize the solutions for existence and uniqueness of a solution to this linearized system.

Proposition 2. *There exists a solution linear equation system (33)-(35) in which all three major macroeconomic variables (y_t^e, i_t, π_t) as serially uncorrelated processes. If all the inverse eigenvalues of the matrix A lie outside the unit circle this solution is unique.*

Proof. For illustration consider π_t analogous arguments can be made for the other two variables. As the system is ergodic the autocovariance generating function is symmetric so $Cov(\pi_t, \pi_{t-l}) = Cov(\pi_t, \pi_{t+l}) \forall l$ so it suffices to show that $Cov(\pi_t, \pi_{t-l}) = 0$ for arbitrary lag length. From the white noise error assumption we know that $E_t u_{t+k}^\pi | \mathcal{I}_t = 0 \forall k > 0$ this means that current inflation can be expressed a function of only contemporaneous shocks $\pi_t = \zeta_\pi^1 u_t^1 + \zeta_\pi^2 u_t^2 + \zeta_\pi^3 u_t^3$. Applying the same argument to period $t-l$ implies π_{t-l} is a function of time $t-l$ errors. Finally the white noise assumption means that $\forall i \forall l E(u_{t-l}^i u_t^i) = 0$ so $Cov(u_{t-l}^i, u_t^i) = 0$. Now the result follows from noting that $Cov(\pi_t, \pi_{t-l}) = \sum_{i=0}^3 \sum_{j=0}^3 Cov(u_t^i, u_{t-k}^j)$ where every term in the summation is zero. \square

The no persistence result arises because the model lacks either intrinsic or extrinsic persistence. It lacks intrinsic persistence because the current value of the state variables X_t can be written as a function of just the current and future values of the shock process u_t independent of their past realizations.²⁴ It lacks extrinsic persistence because the errors are observationally equivalent to expectation errors which means they cannot be persistent. It will carry over to forward-looking policy rules that contain future expected inflation and output gap terms because they can be collapsed into the contemporaneous form (33) as the expectation of future variables will all be zero although verifying the existence condition on the eigenvalues will likely require a numerical computation routine. Appendix A generalizes these results to the richer Generalized Taylor price setting framework and models with capital.

The absence of persistence in the New Keynesian model contrasts with RBC models where the relevant solution variable is simply output y_t - which can inherit persistence from technology or news about future productivity developments Kydland and Prescott [1982] Beaudry and Portier [2006] Beaudry and Portier [2007] Jaimovich and Rebelo [2009] Walker and Leeper [2011] Schmitt-Grohé and Uribe [2012b]. Of course output itself inherits this persistence, it moves one-for-one with the efficient output in the corresponding RBC model so that the output gap stays constant. This highlights the point that Neo-Classical variables and associated shocks do not necessarily appear in the New Keynesian

time series properties from the three core variables in the model. Consult Agro [1995] Bottazzi and Secchi [2003] Bottazzi [2004] for modern expositions of this distribution class. It is sometimes called the Subbotin family of densities after its creator or the Generalized Normal Distribution Version 1.

²⁴In the frequency domain this corresponds to the model acting as a neutral filter preserving the correlation spectrum in the error terms- a point first made by Cogley and Nason [1995].

solution for business cycle dynamics. I discuss this point further in the optimal policy Section 5. Therefore, novel New Keynesian features are needed to improve the fit of the New Keynesian model. It is for this reason that this paper introduces a new feature first-order price dispersion, unique to environments with price rigidity, to the basic New Keynesian model.

The econometric implications are unfortunate. The model cannot be used for forecasting. The key macroeconomic policy variable inflation is white noise. Neither can the model contribute to forecasting output or interest rates- over and above purely statistical procedures or classical models of the natural rate. To confirm: the New Keynesian model is not yet useful for policy. This is particularly unfortunate as the ability to forecast short-term fluctuations is the yardstick against which New Keynesian macroeconomists ask to be judged, as the following quote makes clear:

"We focus on forecastable movements in our variables as we because it is arguably that these constitute the essence of what it means for a variable to be 'cyclical'"

In the articles conclusion (p87) the authors make their point more strongly rejecting a suite of Real Business Cycle models on the following grounds: "We have demonstrated that the forecastable movements in output, consumption and hours [the three main variables in the Real Business cycle framework]"- what we would argue is the essence of the 'business cycle'- are inconsistent with a standard growth model disturbed solely by random shocks to the rate of technical progress." Rotemberg and Woodford [1996] (p71)²⁵ New Keynesian economics is not living by its professed econometric standards. For those with sufficient perspective this is all rather reminiscent of the evolution of Classical economists attitudes towards econometrics encapsulated in the following quote by Nobel laureate Thomas Sargent about fellow laureates Edward C (Ed) Prescott and Robert (Bob) Lucas: "My recollection is that Bob Lucas and Ed Prescott were initially very enthusiastic about rational expectations econometrics. After all, it simply involved imposing on ourselves the same high standards we had criticized the Keynesians for failing to live up to. But after five years of doing likelihood ratio tests on rational expectation models. I recall Bob Lucas and Ed Prescott both telling me that those tests were rejecting too many good models." Sargent [2005]

Worse still the basic model can not even be estimated with data from the three Q_t variables because it is not *identified* in the sense of Lehmann and Casella [1998] or Dufour and Hsiao [2008].

Remark 1. *The structural parameters of the model defined by (33)-(35) are not identified.*

Proof. Applying proposition 2 simplifies the system to:

$$\begin{aligned} y_t^e &= \frac{1}{\sigma}(i_t - \bar{r}) + u_t^1 \\ i_t &= \bar{r} + a_\pi \pi_t + a_y y_t^e + u_t^2 \\ \pi_t &= \omega y_t^e + u_t^3 \end{aligned}$$

Define the vector of endogenous variables $Q_t = (y_t^e, i_t, \pi_t)$ and the parameter vector $\theta = (\gamma, a_\pi, a_y, \omega, \beta, \lambda)$ where $\gamma = \frac{1}{\sigma}$ and λ is the collection of parameters

²⁵see also the abstract of Blinder and Fischer [1981] for a similar definition

governing the joint distribution of the three error terms. Θ denotes the sample space of the parameters formed of the product space $(\Gamma \times A_\pi \times A_y \times \Omega \times B \times \Lambda)$ where for example A_π is the set of admissible values for the parameter a_π . In the common case of normally distributed errors and unconstrained optimization $\Theta = \mathfrak{R}^{11}$ with Λ consisting of the distinct terms of the variance-covariance matrix of the error terms.

The crucial object is $f_{\theta_0}(Q_t)$ the joint probability distribution induced by the particular parameter vector $\theta_0 \in \Theta$ at time t . Recall that a parameter θ is identified when there is a one-to-one mapping to the probability distribution $\theta \rightarrow f_\theta(Q_t)$ at every time t .

Suppose the model were identified and proceed by the counterexample. Since β does not appear in the reduced form I can construct a counterexample with any $(\gamma, a_\pi, a_y, \omega, \lambda) \in (\Gamma \times A_\pi \times A_y \times \Omega \times \Lambda)$ and $\beta_1, \beta_2 \in B$ with $\beta_1 \neq \beta_2$ let $\theta_1 = (\gamma, a_\pi, a_y, \omega, \beta_1, \lambda)$ and $\theta_2 = (\gamma, a_\pi, a_y, \omega, \beta_2, \lambda)$ as $f_{\theta_1} = f_{\theta_2} \forall t$ but $\theta_1 \neq \theta_2$ contradicting the hypothesis of a one-to-one mapping. \square

For concreteness consider the popular Generalized Method of Moments Estimator²⁶ for the structural parameter vector $R_{3 \times 3}$ of the relationship between Q_t and $E_t Q_{t+1}$. It is necessary to have an $m \times 1$ $m > 3$ vector of available instruments $Z_t \in \mathcal{I}_t$ consistent with the orthogonality condition and associated estimator:

$$E_t[Z_t'(Q_t - Q_{t+1}R)] = 0$$

To allow for the case of over-identification where the number of potential instruments in Z_t exceeds the number of moment conditions $m > 3$ in the basic model I minimize the quadratic form of the orthogonality conditions $H_T(R) = [T^{-1}(Z_t'(Q_t - Q_{t+1}R))W_T(Z_t'(Q_t - Q_{t+1}R))']$. Here W_T is a weighting matrix dependent on θ that will turn out to be inversely proportional to the variance-covariance matrix of the orthogonality conditions, as previously T is the number of time observations. Optimization with continuous differentiability in R yields the GMM estimator²⁷

$$\hat{R} = (Q'_{t+1}Z_tW_TZ_t'Q_{t+1})^{-1}Q'_{t+1}Z_tW_TZ_tQ_t$$

with $\hat{R} = (Q'_{t+1}Z_t)^{-1}Q'_{t+1}Q_t$ corresponding to the just-identified case where the orthogonality conditions are solved exactly.

However, applying proposition 2 again shows that this estimator is not defined because $Q'_{t+1}Z_t = 0$ meaning that the first matrix is non-invertible. This follows because Q_{t+1} is comprised entirely of expectation errors which must be uncorrelated with any variable belonging to the information set at time t , \mathcal{I}_t to which Z_t belongs. Hence there are no valid instruments for the expectations of future macroeconomic variables in this system. This implies the structural parameters θ are not identified. For proof suppose the converse that θ were identified (i.e. there were sufficient valid instruments) assumption 2 would bound the expected deviation from the orthogonality conditions which would be sufficient to invoke Hansen and Singleton [1982] to prove weak convergence (in probability) of the

²⁶The approach was developed by Hansen [1982] applied to rational expectations modeling by Hansen and Singleton [1982] and based upon method of moments estimation procedure first employed by Karl Pearson.

²⁷Consult Hansen and Singleton [1982] or a textbook such as Hamilton for a full exposition

GMM estimator $\hat{\theta}_T \rightarrow \theta^{28}$. Now consider the solution for the reduced form parameters in terms of their structural counterparts.

$$\begin{aligned}
R_{11} &= \frac{1}{1 + \gamma(a_y + a_\pi)} > 0 \\
-\infty < R_{12} &= \frac{\gamma(a_\pi\beta - 1)}{1 + \gamma(a_y + a_\pi)} < \infty \\
R_{13} &= 0 \\
R_{21} &= \frac{a_y + a_\pi\omega}{1 + \gamma(a_y + a_\pi)} > 0 \\
R_{22} &= 0 \\
-\infty < R_{23} &= \frac{a_y\gamma(a_\pi\beta - 1) + a_\pi[\omega\gamma(a_\pi\beta - 1) + \beta(1 + \gamma(a_y + a_\pi\omega))]}{1 + \gamma(a_y + a_\pi)} < \infty \\
R_{31} &= \frac{\omega}{1 + \gamma(a_y + a_\pi)} > 0 \\
R_{32} &= 0 \\
-\infty < R_{33} &= \frac{\omega\gamma(a_\pi\beta - 1) + \beta[1 + \gamma(a_y + a_\pi\omega)]}{1 + \gamma(a_y + a_\pi)} < \infty
\end{aligned}$$

Note that each reduced form parameter R_{ij} is a composite of continuous functions and is therefore a continuous function of the structural parameters θ see Aliprantis and Kim. Denote this function by F_{ij} so $F_{ij} = R_{ij}$. Therefore by the continuous mapping theorem of Mann and Wald [1943] $\lim_{T \rightarrow \infty} \hat{F}_{ij} = R_{ij}$ in probability. This would create a one-to-one mapping between reduced-form parameters and probability distributions over the observables Q_t via the probability limits of the reduced form, the probability limits of the structural parameters and the structural parameters themselves. Hence the reduced form parameters would be identified- a contradiction. Therefore the structural parameters must be unidentified.

The notion that the New Keynesian Phillips curve might be weakly identified is not new Woodford [1994] Mavroeidis [2004]. Although I am unaware that the possibility of having *no identification* in the New Keynesian model has ever been set out quite so clearly. In the next section I present a novel test of this unidentified New Keynesian Phillips curve. I document the strongest evidence yet of inflation persistence- which I use as a stylized fact in subsequent model building.

3 Price Dispersion, Approximations and Welfare

The crucial difference between a stochastic and a non-stochastic steady state lies in the behavior of price dispersion. I consider a general setting which can feature both continuous and discrete shock processes. I find that price dispersion which is absent in a non-stochastic model, will always be present in a plausible stochastic environment.

²⁸

In Subsection 3.1 I define a concept of *inefficient price dispersion* that can arise in New Keynesian models. I then prove that it will arise in all but a few highly stylized New Keynesian models. I then characterize its dynamic behavior in general and in the two most popular variants of the New Keynesian models those with Calvo and Taylor contracts. Although, not conceptually difficult to prove and extremely general in scope these results overturn existing thinking. For example, the following quote in which the authors seem to believe that New Keynesian models cannot generate price dispersion unless there is a non-zero rate of trend inflation. A claim which I prove to be false.

"... many New Keynesian models such as those in Clarida et al. [1998] or Woodford [2011a], generate price dispersion if and only if there is inflation ... But the data suggests there is price dispersion during periods of zero or low inflation (something first noted in Campbell and Eden [2014].) This suggests it is important to work with (non-New Keynesian) models that can deliver price dispersion without inflation." Head et al. [2012] (p. 942)

In Subsection 3.2 I apply these results to several popular topics in New Keynesian modeling including zero lower bound, stochastic volatility and regime switching models. In each case I am able to show that major papers are not just mathematically in error but that their results provide a qualitatively misleading account of macroeconomic dynamics under the relevant mechanism. I discuss the (often adverse) implications for these models ability to match empirical evidence.

Subsection 3.3 characterizes price dispersion as a random variable in relation to underlying stochastic processes in the economy. In application these results invalidate the basic New Keynesian Phillips curve and its forward solution, equations (24) and (32). Subsection 3.4 shows that inefficient price dispersion constitutes a new form of market failure. I show how it originates on the *producer side* of the economy and link it to the failure of Acemoglu's representative firm theorem Acemoglu [2008] I demonstrate that it cannot be corrected by tax policy even when unrealistic lump sum tax and subsidy combinations were allowed.

In Subsection 3.5, I link topological conjugacy to the Lucas critique and show that representing a New Keynesian model by a log-linear approximation about the non-stochastic steady state induces a failure to meet the Lucas critique. This is a very serious charge against existing New Keynesian models for which passing the Lucas critique is a *raison d'être*. Finally, in Subsection 3.6 derives a correct notion of *stochastic equilibrium* which exists in all New Keynesian models. The equilibrium concept could be applied elsewhere in economics but I do not pursue this point here. I use the mathematical concept of *topological conjugacy* to prove why it is legitimate to represent the dynamics of the economy close to equilibrium by taking a log-linear approximation about the center of this distribution from which I derive a correct Phillips Curve.

3.1 Non-Stochastic Environments

The following powerful result derived directly from the construction of the price level tells us that the measure of price dispersion Δ defined by equation (18) in Section 2 is strictly greater than unity unless all firms set the same price.

Lemma 2. $\Delta \geq 1$ with $\Delta = 1$ if and only if $p_t(i) = P_t, \forall i$.

Where $p_t(i)$ is the price set by any firm i at time t . The lengthy proof is contained in Appendix A the first part is a familiar application of Jensen's inequality, possible because the demand system is sufficiently convex and the second a small extension exploiting strict convexity. The extension to other New Keynesian models that use the constant elasticity of substitution preference scheme is simple. I do so in Appendix A by modifying the probability measure used to aggregate the various prices to obtain the price level to correspond to three common pricing models the basic Calvo model used here, the Calvo model with indexation to trend inflation used by Yun [1996] and the General Taylor Economy of Taylor [1993b] Coenen et al. [2007] Dixon and Kara [2010] Dixon and Kara [2011] Dixon and Le Bihan [2012] which encompasses a wide range of pricing models and can be fitted exactly to match cross-section price distributions.²⁹ The result is not specific to one demand systems. To allow heterogeneity between firms it is necessary to define a concept of *efficient* price dispersion. Efficient price dispersion measures the price dispersion that would exist in the corresponding flexible price model- where all prices could be reset in every period at no cost with perfect information- as in the RBC framework. It takes the form

$$\Delta_t^* = \int_i F_{i,t} \left(\frac{p_{i,t}^{**}}{P_t} \right) d\mu_{i,t} \quad (41)$$

where $P_t = \int_i p_{i,t} F_{i,t} \left(\frac{p_{i,t}^{**}}{P_t} \right) d\mu_{i,t}$ is the price level. $F_{i,t}$ represents the demand curve for firm i and is homogeneous of degree zero in prices- as in basic consumer theory. This allows us to define *inefficient* price dispersion as the ratio between *actual* and *efficient* price dispersion.

$$\Delta_t = \frac{1}{\Delta_t^*} \int_i F_{i,t} \left(\frac{p_{i,t}}{P_t} \right) d\mu_{i,t} \quad (42)$$

This definition is consistent with the parametric form of Δ_t in the Calvo model given in (18) where I assumed firms faced the same demand and technology with no idiosyncratic shocks, which makes any price dispersion inefficient. As before i is a number used to index an individual firm. Since I am allowing heterogeneity among firms' optimal prices and the possibility for multiple equilibrium, it is necessary to be more precise about how i is assigned. i reflects the order of the firm in the price distribution. Therefore, there exists a positive monotonic relationship between i and $\frac{p_{i,t}}{P_t}$. This also simplifies the existence of the defining integral.³⁰ $\Omega_{i,t}$ is the set of all prices in the economy at time t . Σ_t is the family of sets of individual firms over which output can be aggregated.

²⁹Dixon [2012] shows that the Generalized Taylor model can approximate arbitrarily well the Generalized Calvo used by authors such as Wolman [1999] Dotsey and King [2006] Sheedy [2010], the multiple Calvo associated with Carvalho [2006] de Carvalho [2011], as well as the familiar simple Taylor and Calvo.

³⁰Formally I have defined that Σ_t contains a countable family of pure points representing firms interacting strategically together and another family of Borel sets which are continua of firms who take the aggregate economy as given corresponding to perfect or monopolistic competitions as in Calvo, Taylor and other New Keynesian models. The associated measure μ_i is positive as it reflects shares of goods in aggregate consumption- the existence of a discrete Lebesgue integral over the pure point sets follows immediately. Royden and Fitzpatrick proves that monotone functions on Borel sets in \mathcal{R} possess Lebesgue integrals. This demonstrates that a measure μ_t exists over all sets in Σ_t .

Hence $p_{i,t}^{**}$ represents the price that a firm at the corresponding point in the price distribution would set if *all* prices were flexible and information perfect as in the RBC framework. This differs from p_t^* which features in staggered price setting models such as the Calvo model in Section 2 and Taylor contracting models in this section, where p_t^* represents the price a firm with a price that is fully flexible today would set taking into account that other prices in the economy are rigid. The notion that rigidity of prices elsewhere in the economy causes flexibly reset prices to differ from those that would be set in a fully flexible world ($p_{i,t}^{**} \neq p_{i,t}^*$) is known as *real rigidity* see Ball and Romer [1990]. It is present in major New Keynesian models. For example in the Calvo model from Section 2- the relevant equation is (16). The RBC skeleton has a symmetric equilibrium so $p_t^{**} = P_t$ however the optimal reset price p_t^* only equals P_t when $\pi_t = 0$ otherwise $p_t^{**} \neq p_t^*$ so there is real rigidity. It would be interesting to explore the link between these two concepts further. To complete the generalization it is necessary to define two properties that DSGE models or more specifically their associated price distributions may possess. The first is *aggregate nominal rigidity*

Definition 1. An economy possesses aggregate nominal rigidity if there exists a measurable set of firms $i \in \mathcal{B}^1 \in \Sigma_{i,t}$ such that for some $l > 0$ $p_{i,t} = \Phi_i(p_{i,t-l}, \cdot)$ and there exists an inflation rate $\bar{\pi}(\sigma_t)$ such that $\pi_t \neq \bar{\pi}_t$ implies that $p_{i,t} \neq p_{i,t}^{**}$ and $\bar{\pi}(\{\emptyset, p\})$. Also it must be the case that $\forall l'$ where $0 < l' < l$ we can write $p_{i,t-l'} = \Phi_{i,l-l'}(p_{i,t-l})$ where $\pi_{t-l'} \neq \bar{\pi}(\sigma_t)_t$ implies $p_{i,t-l'} \neq p_{i,t-l'}^{**}$

The first part states that to have aggregate price rigidity there must be a positive fraction of output sold at a price that reflects past prices l periods back and differs from those that would prevail in the flexible economy. There is an allowance that if inflation hits a certain value the two could coincide as would occur in Calvo contracting starting from no price dispersion starting from no price dispersion when inflation is zero or equal to a target $\bar{\pi}$ to which all prices are indexed either directly as in Yun [1996] or to the last periods inflation as in Smets and Wouters [2003] Christiano et al. [2005] Smets and Wouters [2007]³² The second part serves to ensure dependence between past and current price levels and therefore past and current levels of price dispersion. This means there cannot be aggregate price rigidity in an otherwise flexible economy just because there are backward-looking or cycling prices. This restriction has economic content. Many items are sold on temporary discount which upon expiry return to their old level. Fortunately, recent New Keynesian models that feature products on sale avoid this trap as they are able to generate changes in the frequency or size of discounts in response to monetary shocks that would be neutral in the models RBC skeleton- which implies purchases are made at

³¹In mathematical terms Ω_t is the measurable space of firms and Σ_t is the smallest sigma-algebra which contains sets of firms that produce positive output and the empty set with no firms in it. The distinction between membership of Σ_t and Ω_t is operative in the specification of many macroeconomic models with imperfect competition which use continuum of firms, for which countable subsets of prices would belong to Ω_t but not Σ_t . The two coincide in real life where the set of prices at a point in time is countable.

³²With Taylor contracts we have to allow for the possibility that there may exist reset prices consistent with no inefficient price dispersion that differ over time. This is because prices replace one another under Taylor contracting see equation (49) so the price that sets Δ_t to one will depend on the price it is replacing in that case p_{T-M}^* .

$p_t \neq p_t^{**}$ see Kehoe and Midrigin [2008]Guimaraes and Sheedy [2011]Nakamura and Steinsson [2011]Eichenbaum et al. [2011]Malin et al. [2015].³³

As well as traditional RBC models, the definition of aggregate price rigidity excludes recent New Monetarist models. In this framework there is a distribution of prices motivated by a flexible price microeconomic model, it has become common to use a model with costly search such as Burdett et al. [1983] Albrecht and Axell [1984] Burdett and Mortensen [1998] as these can provide a rational for agents to hold money if there are appropriate credit constraints see Lagos and Wright [2005]Williamson et al. [2010]. Their point is that provided the distributions of prices overlap from period to period it is possible for some firms not to change their price in equilibrium. Even though money will be exactly neutral because the market equilibrium in their model does not depend on the money supply. They claim therefore that price rigidity does not imply monetary non-neutrality.

Their claim is completely correct. However, this is because their model does not possess aggregate nominal rigidity. It has equilibria where individual firms choose to keep prices fixed because they do not care about their position in the price distribution by equilibrium construction. On the other hand, with my approach of indexing firms by their position in the distribution there is no nominal rigidity. At each point in the distribution the appropriate firm raises their price one-for-one with the money supply- so the aggregate price level is perfectly flexible.

It is necessary to impose one further condition on the price distribution this is called *nominal heterogeneity*. This states that the degree of nominal distortion represented by the ratio between the actual price and the flexible model price $p_{i,t}/p_{i,t}^{**}$ must vary between firms.

Definition 2. An economy possesses aggregate nominal heterogeneity if $\exists \mathcal{B}^\infty, \mathcal{B}^\epsilon \in \Sigma_{i,t}$ such that $\int_{\mathcal{B}^\epsilon} p_{i,t}/p_{i,t}^{**} d\mu_i - \int_{\mathcal{B}^\infty} p_{i,t}/p_{i,t}^{**} d\mu_i > 0$

This restriction rules out stylized models such as Barro [1972]Sheshinski and Weiss [1977] Rotemberg [1982]Mankiw [1985] where all firms set the same price motivated by physical costs to price changing that do not differ among firms, as argued earlier such models are often unable to generate sufficient nominal rigidity. One of their creators refers to them as "toy models" see FIND REFERENCE In any case physical costs of price changing vary substantially across firms and products see Levy et al. [1997]- consistent with observed heterogeneity in the frequency of price adjustment as found in empirical studies such as Dhyne et al. [2006]Dickens et al. [2007]Dixon and Le Bihan [2012].

Note that *efficient* price dispersion is a *relative* concept of efficiency. It compares the actual distribution of prices to a corresponding model with flexible prices, perfect information and profit maximization³⁴. The price dispersion that is efficient from the firms' point of view Δ_t^* could be inefficient from a social planner point of view because there are other inefficiencies in the economy (e.g.

³³Note that as these papers do not tend to use log-linearization around a non-stochastic steady state to characterize business cycle dynamics. They are immune to the subsequent criticisms of this section.

³⁴With suitable modification of the RBC skeleton profit maximization could be relaxed to firm objective maximization to allow for among other factors risk aversion or behavioral factors as considered by for example Jaimovich and Rebelo [2007] Choudhary and Levine [2010], provided it did not induce direct dependence between today's optimal price and past optimal prices conditional on other shocks and parameters in the model.

imperfect competition or asymmetric information) that cause welfare to fall below its social optimum. In fact it could be constrained (second-best) efficient to have inefficient price dispersion in order to help mitigate other uncorrected externalities. A prominent attempt to demonstrate this point is the burgeoning optimal inflation rate literature, where it is common to augment benchmark New Keynesian models capable of generating (inefficient) price dispersion, such as the basic Calvo model, with additional frictions in order to derive non-zero optimal inflation targets.³⁵ In fact, as inflation implies inefficient price dispersion³⁶, all existing attempts to resolve the 'Divine Coincidence' in Section 4 can be viewed through this lens also. Although, I show existing calculations are not correct.

Definition 3. z

$$z \tag{43}$$

³⁷. I could even alter the source of the price dispersion from staggered to for example information-constrained price-setting Mankiw and Reis [2002]Mankiw and Reis [2006]Mankiw and Reis [2007]Lorenzoni [2009]Lorenzoni [2010]Nimark [2008]Nimark [2014]?Adam [2007]Mackowiak et al. [2008]Paciello and Wiederholt [2014] are papers where this price dispersion is present but not accounted for. All that is required is a motivation for firms to set different prices when in a flexible price world it would be efficient if they all set the same.³⁸The behavior of price dispersion and its dynamics are integral to all the analysis which follows. All results that do not refer to a specific specification of sticky price setting (e.g. Calvo or Taylor) generalize to all models covered by this lemma.

³⁵The first friction studied was the existence of non-interest bearing money which brought the deflationary forces of the Friedman rule into place see Khan et al. [2003]Adão et al. [2003]Schmitt-Grohé and Uribe [2004]Schmitt-Grohe et al. [2007]. Subsequently, further imperfections in the product, goods and labor markets have been considered Collard and Dellas [2005]Pontiggia [2012]Ikeda [2015], along with a binding lower bound on nominal interest rates Billi et al. [2011]Coibion et al. [2012]Eggertsson and Giannoni [2013]Eggertsson et al. [2015]. As this literature only considers price dispersion owing to trend inflation and ignores the additional dispersion created by *stochastic* shocks there will be an upward bias in reported optimal inflation. It is beyond the scope of this paper to quantify this omission.

³⁶This follows from Theorem 1. The example in Appendix B.2.1 where inflation corrects initial price dispersion does not apply to a Divine Coincidence framework as detailed in Section 4. Many of these 'second-best' considerations could arise in a model with stochastic price dispersion and would constitute interesting extensions of this paper. Although, as I characterize optimal policy in Section 5 by a different welfare standard- results would not be directly comparable.

³⁷To allow for heterogeneity between firms we would have to redefine Δ to normalize each firm's price relative to its optimal reset price. Yun and Levin [2011]Fuhrer [2000]Dennis [2009]Ravn et al. [2010]Givens [2013]Santoro et al. [2014]Lewis and Poilly [2012]Lewis and Stevens [2015]Etro and Rossi [2015]consider various alternative demand systems with a variety of motivations.

³⁸Price dispersion would also come about where there are physical costs of price changing provided that firms face idiosyncratic shocks or differing adjustment costs. See for example Gertler and Leahy [2008]Nakamura and Steinsson [2008]Reiff et al. [2014]Bouakez et al. [2009]Bouakez et al. [2014]. In these cases the relevant interpretation of the price dispersion variable Δ is the difference between the actual and flexible price. Suppose for example fixed adjustment costs of a price change varying across firms with an aggregate shock- $\Delta > 1$ will come about if some firms' adjustment costs are below and some above the common adjustment threshold. Similarly, with common fixed adjustment cost but idiosyncratic shocks $\Delta > 1$ will occur if some firms keep their price constant because their idiosyncratic shock 'cancels out' the aggregate shock i.e. they remain inside their band of price inaction.

The fundamental mechanism in this paper is that inflation causes price dispersion. The most general statement that can be made is as follows. It assumes away nominal indexation. Consult Appendix B for an analogous result with nominal indexation in place.

Theorem 1. *With non-trivial price rigidity if $\pi_t \neq 0$ then $\exists t' \leq t$ such that $\Delta_{t'} > 1$*

Proof. The result is trivial if $\Delta_{t-1} > 1$ so assume $\Delta_{t-1} = 1$. By hypothesis that $\pi_t \neq 0$ it follows that $P_t \neq P_{t-1}$. Now we know from lemma 2 that $\Delta_{t-1} = 1$ implies that $\forall p_i \in \Omega_{t-1} p_i = P_{t-1}$. Therefore $\pi_t \neq 0$ requires that $\exists p_j \in \Omega_t$ such that $\exists p_j \neq p_i$. The assumption of non-trivial price rigidity ensures that there exists $p_k \in \Omega_{t-1}, \Omega_t$ from the first part $p_k = p_i$, therefore from the second part for some $p_j \in \Omega_t p_k \neq p_j$ so by lemma 2 $\Delta_t > 1$. \square

Note that this general result cannot be tightened to link inflation to *contemporaneous* price dispersion because this would not encompass models with Taylor contracts. The reason is that with Taylor contracts the price required to remove price dispersion can differ from that required to stabilize prices (zero inflation). This is because under Taylor contracts inflation is determined by a comparison between current reset prices and those they are replacing whilst price dispersion is determined by the difference between current reset prices and past reset prices that have *not* been replaced. Intuitively, non-zero current inflation can cancel out past price dispersion. Under Calvo where old reset prices never disappear the zero price dispersion and zero inflation reset price coincide so the coexistence must be contemporaneous. Appendix A offers a simple numerical example to clarify these points. Indeed, price dispersion persists even if the shock process generating it is not present in all time periods. The most general statement can be made in the context of the benchmark Calvo model set out in Section 2.

3.2 Price Dispersion with Calvo Pricing

Remark 2. *If inflation π_t is ever non-zero in the Calvo model price dispersion $\hat{\Delta}_t > 0$ will exist in all subsequent periods.*

The result follows simply from applying Lemma 2 and noting that the set of prices in the economy Ω_t includes every previous price. This is because the fraction of prices in the economy equal to a given reset prices p_t^* never falls to zero matter how far into the future one moves since $\iota_T(p_t^*) = \alpha^{T-t}(1 - \alpha) > 0 \forall T > t$. This result has powerful implications for the class of equilibrium that can exist in a model with Calvo pricing. In particular it implies the current concept of equilibrium used in the literature the non-stochastic equilibrium does not exist in a Calvo model if it has ever had price dispersion.

Definition 4. The behavior of the New Keynesian model from time t can be represented³⁹ by the continuation path $\mathcal{Z}_t^C = \langle Z_t, Z_{t+1}, \dots \rangle$ where $Z_t = (\pi_t, y_t^e, \Delta_t)$ which are governed by all the conditions set out in Section 2 apart from the policy rule equation (16).

³⁹The arguments in Section 2 demonstrate a topological conjugacy to between Z_t and the complete model- this topic is discussed in more detail in Section 3.6.

The specific policy rule equation (16) is omitted to allow consideration of the constraints on policy rules in general. Here policy is represented implicitly by continuation paths $\{\pi_t^C, y_t^C\}$ - policy rules comparable to (16) but possibly time dependent could then be derived using the Euler equation (7).⁴⁰

Definition 5. A non-stochastic continuation path from time t denoted $\bar{Z}_t^C = \langle \bar{Z}_t, \bar{Z}_{t+1}, \dots, \bar{Z}_{t+\tau}, \dots \rangle$ is where $\forall \tau \geq 0 \Pr(Z_{t+\tau}) = \bar{Z}_{t+\tau} = 1$, i.e. there is no uncertainty about future variables.

A *non-stochastic continuation path* corresponds to a perfect foresight model with initial value Z_t . As Z_t is dependent upon the shock carrying parameters from Section 2 $\Theta_T = (\psi_T, \varphi_T, A_T)$ in a continuous fashion. The perfect foresight applies to the continuation of the shock processes also⁴¹ $\bar{x}_t^C = \bar{x}_t^C = \langle \bar{\Theta}_t, \dots, \bar{\Theta}_{t+1}, \dots, \bar{\Theta}_{t+\tau}, \dots \rangle$ so $\Pr(\Theta_{t+\tau}) = \bar{\Theta}_{t+\tau} = 1 \forall \tau \geq 0$. A stronger concept is that of a *stable non-stochastic continuation path* defined as follows.

Definition 6. A stable non-stochastic continuation path denoted $(\bar{Z}^*)_t^C$ is a non-stochastic continuation path where the shock process are held constant $\Pr(\Theta_{t+\tau}) = \bar{\Theta} = 1 \forall \tau \geq 0$

This corresponds to a non-stochastic model with initial position Z_t . Finally the strongest concept is that of non-stochastic equilibrium path from t .

Definition 7. A non-stochastic equilibrium path from t of a New Keynesian model denoted $(\bar{Z}^{**})_t^C$ is a non-stochastic continuation path where $Z_{t+\tau} = \bar{Z} \forall \tau \geq 0$ i.e. all the variables remain constant in all future periods for sure.

In other words a non-stochastic equilibrium is a fixed point of the system where every future variable is certain to be constant at its present period value forever. This is the basic solution concept in microeconomics and growth theory. It is natural macroeconomists wish to apply it to the New Keynesian model also. However, whenever price dispersion is possible this is not in general correct even in the extreme case of a non-stochastic continuation path⁴².

Definition 8. $Z_t^H = \langle \dots, Z_{t-1}, Z_t \rangle$ denotes the history of the variable Z up to time t

$$\text{Let } \Delta(\bar{\pi}) = \Delta(Z_t^H = \langle \dots, \bar{\pi}, \bar{\pi} \rangle)$$

Proposition 3. *In a model with Calvo pricing, a non-stochastic equilibrium $(\bar{Z}^{**})_t^C$ with $\pi_T = \bar{\pi}$ from time t will only exist if $\Delta_t = \Delta(\bar{\pi})$.*

⁴⁰For convenience I use raw output y_t rather than the efficient output gap y_t^e introduced in Section 2 to characterize policy. The two formulations are equivalent as however the output gap variable is defined there must be a one-to-one mapping between them in a non-stochastic world. I suspend discussion of what is a good definition of the output gap for the empirically relevant case of a stochastic model in Section 6(??).

⁴¹Formally there is a non-stochastic continuation path for Z_t if and only if there is a non-stochastic equilibrium for Θ_t with the *only if* following from the continuous dependence and the *if* following from the fact that Θ_t is the only source of uncertainty in the model.

⁴²Formally, a non-stochastic continuation path is necessary but not sufficient for a non-stochastic equilibrium. The necessity follows from noting that otherwise expectation errors could lead actual and expected values to diverge so that $\Pr(Z_{t+\tau}) = \bar{Z} < 1$ The non-sufficiency follows from the following counter-example.

Proof. The argument proceeds by contradiction suppose the converse then (21) takes the form of the following deterministic difference equation.

$$\Delta_T = \Delta(\bar{\pi}) + \vartheta^{T-t}(\Delta_T - \Delta_t) \quad (44)$$

Where

$$\vartheta = \alpha(1 + \bar{\pi})^\theta$$

$$\Delta(\bar{\pi}) = (1 + \bar{\pi})^\theta \frac{(1 + \bar{\pi} - \alpha)^{\frac{\theta}{\theta-1}}}{(1 - \alpha)^{\frac{1}{\theta-1}}(1 - \alpha(1 + \bar{\pi})^\theta)}$$

$\Delta(\bar{\pi})$ is the non-stochastic steady state price dispersion and $\vartheta < 1$. This restriction is a requirement to ensure that a steady-state exists if not Δ would grow without bounds which would cause consumption C to tend to zero violating the transversality condition, equation (6).⁴³ Now it is clear that if $\Delta_t \neq \Delta(\bar{\pi})$ it means Δ_T is time dependent contradicting the definition of a non-stochastic equilibrium for Z from time t . Note also that as $0 < \vartheta < 1$ the economy is converging monotonically towards its non-stochastic steady state but does not reach in finite time. Therefore we know that $1 < \Delta_T < \max\{\Delta_t, \Delta(\bar{\pi})\}, \forall T > t$ \square

For the zero-inflation steady state the behavior of Δ simplifies considerably to

$$\Delta_t = 1 - \alpha + \alpha\Delta_{t-1}$$

$$\Delta_t = \alpha^{t-t_0}\Delta_{t_0} > 1 = \bar{\Delta}$$

In other words the persistent behavior of the backward-looking price dispersion term stops the forward-looking variables output gap and inflation reaching equilibrium. Note however that the effect of initial price dispersion decays away when inflation is kept constant since $\lim_{T \rightarrow \infty} \Delta_T = \Delta(\bar{\pi}) \forall \Delta_t$. The (non-linear) Phillips Curve relation corresponding to (24)? ensures y_t also has a limit (by the open mapping theorem once again) thus with a stable inflation policy the economy is ergodic $\lim_{t \rightarrow \infty} Z_T = \bar{Z} \forall Z_t$. The property that in the infinite limit a dynamical system "forgets" its initial position is called ergodicity. In sub-section 3.3(?), the stochastic analogue of this concept will be used to define a meaningful notion of dynamic stochastic general equilibrium (DSGE). THEOREM TRADE OFF IN HERE GO FOR MATHS TRADE OFF ENDOGENOUS PERSISTENCE

BOLLOCKS!! Take $t > t_0$. Since $\pi_t = \bar{\pi}$, $y_t = \bar{y}$ is not in fact an equilibrium of the system because it does not conform with all the optimization and market clearing conditions that define the dynamical system laid out in Section 2. To see this note that price construction equation (16) implies a each π_t maps to only one reset price $\frac{p_t^*}{P_t}$ we can see this because the relationship between the optimal reset price and inflation is strictly monotonic- as $\frac{d\pi_t}{d(p_t^*/P_t)} = \frac{1-\alpha}{\alpha} \frac{p_t^*/P_t^{-\theta}}{1+\pi_t} > 0$ Therefore the reset price will be constant in every period $\frac{p_t^*}{P_t} = \frac{\bar{p}}{\bar{P}}$. By recursively solving the optimal reset price equation (15). This implies real marginal costs $\varphi_t = \bar{\varphi}$ will also be constant. From the marginal cost expression equation (12) with no technology shocks $A_t = \bar{A}$ the real wage must be constant

⁴³The threshold inflation rate $\bar{\pi} = \frac{1}{\alpha} - 1$. This restriction is well-known in the trend inflation literature. For developed countries it tends to be met quite easily see Ascari and Sbordone [2014]. I assume in this paper that it is always met for sure.

$W_t = \bar{W}$. Now note that in equilibrium the market clearing condition implies when Δ decreases labor L will decrease one-for-one which sets up a contradiction when we consider the optimal labor supply condition equation (8) by assumption the left hand side is constant but the right hand side must be decreasing-which completes the proof. Therefore persistence in the backward looking variable Δ will transmit to the other variables in the model. This mechanism is central to the analysis of the paper. Also note the following:

Corollary 1. *With Calvo pricing, there will always exist a non-trivial trade-off between inflation and output gap stabilization if there has ever been inflation variability.*

I have just shown that $\pi_t = \bar{\pi}$ implies $y_t^e \neq \bar{y} \forall t > t_0$ if there was initial price dispersion $\Delta_{t_0} > 1$ this implies that $y_t^e = \bar{y} \forall t > t_0$ only if $\pi_t \neq \bar{\pi} \forall t > t_0$. The link with inflation variability is provided by Lemma 2 it does not matter when the price inflation variability took place because of the permanence of price dispersion result remark 1.

This is a profound result it demonstrates that in a Calvo model there exists a non-trivial trade-off between inflation and output stabilization in any non-degenerate stochastic environment. The profound theoretical significance of this result is discussed in detail in Sections 4- where its significance to optimal policy is derived.

3.2.1 Price Dispersion with Taylor Contracting

This sub-section extends the results from the Calvo model to the Taylor contracting framework. Several results change but the theme that price dispersion generates staggered adjustment of the economy to shocks is retained. Furthermore, assuming the economy jumps to non-stochastic equilibrium immediately has misleading implications for the behavior of price dispersion with implications for welfare.

There is no analogue of remark 1 with Taylor contracts there is a maximum contract length so all contracts will eventually disappear from the price level-creating the possibility for price dispersion to disappear i.e. $\hat{\Delta} = 0$ if all the reset prices are the same for sufficiently long. Therefore Proposition 3 and corollary 2 apply only for as long as there exists prices set before $T = t_0$ that have not been reset. I focus here on the case of simple Taylor Appendix ? covers the simple extension to Generalized Taylor economy of Dixon and Le Bihan [2012]. Consider a simple Taylor economy where contracts last for M periods. There is staggered price adjustment so a fraction of firms $1/M$ are allowed to reset their price each period. In the knowledge that this price will remain fixed for exactly M periods. Therefore the price level construction equation takes the form:

$$P_t^{1-\theta} = \frac{1}{M} p_t^{*1-\theta} + \frac{1}{M} p_{t-1}^{*1-\theta} + \dots + \frac{1}{M} p_{t-(M-1)}^{*1-\theta} \quad (45)$$

Firms set their reset prices as a weighted average of real marginal costs over the course of the contract so:

$$\max_{p_t^{(i)}} E_t \sum_{T=t}^{t+M} Q_{t,T} \left[\frac{p_t^{(i)}}{P_T} y_T - \varphi_T y_T \right] \quad (46)$$

Changes in the price level reflects the difference between the current reset price p_t^* and the price it replaced p_{t-M}^* .

$$P_t^{1-\theta} - P_{t-1}^{1-\theta} = \frac{1}{M}(p_t^*)^{1-\theta} - \frac{1}{M}(p_{t-M}^*)^{1-\theta} \quad (47)$$

from which can be derived the following expression for inflation

$$(1 + \pi)^{\theta-1} = 1 + \frac{1}{M}\left(\frac{p_{t-M}^*}{P_t}\right)^{1-\theta} - \frac{1}{M}\left(\frac{p_t^*}{P_t}\right)^{1-\theta} \quad (48)$$

Now the evolution equation for Δ analogous to equation (19) in the Calvo model is

$$\Delta_t = \frac{1}{M}(p_t^*)^{-\theta} - (p_{t-M}^*)^{-\theta} + (1 + \pi)^\theta \Delta_{t-1} \quad (49)$$

To see that a trade-off between inflation and output stabilization exists for the first $M - 1$ periods I proceed by contradiction. Assume an equilibrium $Z_t = \bar{Z}$ exists from equation (45) you can see that to have a constant level of inflation $\pi_t = \bar{\pi}$ there must be a one-for-one relationship between p_t^* and p_{t-M}^* . Given $\Delta_{t_0} > 1$ there must be at least one price in the period t price level p_{t-j}^* where $0 < j < M$ such that $p_{t-j}^* \neq p_t^*$. When this price comes to be replaced at $p_{t-j+M}^* \neq p_t^*$ the optimal reset price equation (43) requires that $W_t \neq W_{t-j+M}$. Now by hypothesis $y_t = \bar{y}$ so from equation (8) the real wage and labor supply must move in the same direction- however this means aggregate income has increased- which violates equation (9)- the condition that all income must be consumed. From period $t + M$ onwards the economy reaches non-stochastic equilibrium as $\Delta = 1$. Therefore it takes precisely M periods for the Taylor economy to transition to non-stochastic equilibrium. This equilibrium is efficient i.e. equal to the flexible price output. The non-stochastic system is therefore ergodic. The result extends easily to the Generalized Taylor set-up where the non-stochastic equilibrium is reached after J periods- where J is the length of the longest contract.

3.2.2 Applications

It is common to represent a New Keynesian economy with staggered nominal adjustment as switching between alternate non-stochastic steady states with no dynamic adjustment. This sub-section has proven that this approach is erroneous. The two models to which this approach has been most commonly employed have featured binding zero lower bounds on monetary policy and switches in monetary policy regime. It has also been used to model major exchange rate devaluation and structural adjustment episodes Uribe and Schmitt-Grohe [2012]Schmitt-Grohé and Uribe [2012a]Farhi and Werning [2012]Farhi and Werning [2014]Na et al. [2014]Eggertsson [2012]Eggertsson et al. [2014]. Zero lower bound models seek to operationalize Keynes' idea of a liquidity trap. The idea is that arbitrage with money, which yields a zero nominal interest rate, prevents the Central Bank from cutting nominal interest rates below zero. Therefore if there is a sufficiently large fall in aggregate demand such that the desired nominal interest rate falls below zero. This zero bound will bind such that the economy will be demand-constrained with inefficiently low output. Crucially, in a New Keynesian model because the representative consumer is

forward-looking liquidity traps cannot be permanent or the consumption problem explodes and the transversality condition is violated⁴⁴. The literature on zero lower bound models is voluminous. They have been used to explain large economic contractions following financial crises and study optimal monetary and fiscal policy responses that might mitigate or overcome the constraint of the zero bound on nominal interest rates Corsetti et al. [2010] Lorenzoni and Guerrieri [2011] Eggertsson and Krugman [2012] Farhi and Werning [2013] Corsetti et al. [2013] Correia and Farhi [2013]⁴⁵. Unsurprisingly, such models have become extremely popular following the global financial crisis of 2008 and the subsequent spell of near zero short interest rates across major industrialized economies.

The timing convention in these two steady state models is as follows the economy starts at non-stochastic equilibrium- then there is an unanticipated shock. The shock has to be unanticipated or inflation would fall as the time of the zero bound spell approached in anticipation of the future deflation. To clarify this behavior would not contradict proposition 1- which is derived under the assumption that there is no zero bound on nominal interest rates. Finally leaning on the forward-lookingness derived in proposition 1 the model is closed with the economy jumping back to its non-stochastic steady state. This amounts to assuming the liquidity trap is a one-off (or occurs with vanishingly small probability) but would be valid if the probability of the economy transitioning from the normal-times benchmark equilibrium to a liquidity trap were sufficiently small to make the expected deflation associated with future zero bound spells of an order of magnitude less than or equal to the squared term in the series expansion of inflation⁴⁶. This is wrong! All the above models employ Calvo pricing which means that even if the economy begins at a non-stochastic equilibrium- it will never reach another one either whilst the zero bound is binding or afterwards when the shock has been turned off.

An alternative strategy has been to fix the length of the zero bound spell. The timing convention here is that the economy starts from non-stochastic equilibrium then experiences a shock known in advance to last T periods. The model is again closed with the economy jumping straight back to non-stochastic equilibrium (absent policy changes) when the shock is turned off. There is no steady state whilst the zero bound binds, as reseters at different times face different

⁴⁴In an overlapping generation model with borrowers and savers this result does not apply as demonstrated by Eggertsson and Mehrotra [2014] although a permanent liquidity trap would appear implausible.

⁴⁵See also Woodford [2011b] Eggertsson and Giannoni [2013] Benigno et al. [2014] Denes et al. [2013] Eggertsson and Woodford [2004] Eggertsson [2006] Eggertsson [2011] Eggertsson et al. [2009] Adam and Billi [2007] Werning [2011] Cook and Devereux [2011b] Cook and Devereux [2011a] Cook and Devereux [2013] Araújo et al. [2013] Schmitt-Grohé and Uribe [2014].

⁴⁶Whether this alternative assumption is valid is difficult to gauge for two reasons. First it may be difficult to delineate zero bound spells- as their occurrence may be sensitive to changes in the policy environment- for example Eggertsson [2008] assumes the United States was characterized by a deflationary liquidity trap during the early phase of the Great Depression 1929-1933 on the grounds that under a stabilizing policy regime it would have been. However, in reality short term interest rates were significantly above zero throughout this time- which to me indicates the United States was not in a liquidity trap at this time. Similarly, world war 2 mobilization and financial repression measures designed to ease war financing makes it very difficult to ascertain the model consistent definition of the end of the zero bound spell- which is the point when private sector demand had fully recovered from the Great Depression shock see Reinhart [2012]. Finally, it has proved challenging to explain the length of zero bound spells, simultaneous with the small fall inflation- which calls into the question the validity of any parameter estimates.

length of zero bound spell relative to non-stochastic steady state equilibrium, so deflation will in fact moderate over the course of the zero bound spell. This approach is now the more popular Cogan et al. [2010] Christiano et al. [2011] Amano and Shukayev [2012] Erceg and Lindé [2014] Gertler and Karadi [2011] Gertler and Karadib [2013].

However, the approach in these papers still falls foul of the results in this subsection. The dynamics during the zero bound spells will be wrong because these papers all ignore the effect of the price dispersion associated with the deflationary shock implied by lemma 2 and corollary 1. Secondly, the economy will never return to steady state because of the assumption of Calvo pricing in each of these papers and corollary 2.

Of course many results from these papers would stand up as they do not depend on a particular specification of inflation dynamics. For example, Rognlie et al. [2014] Korinek and Simsek [2014] show that liquidity traps generate deep recessions followed by recoveries even when the extreme old Keynesian assumption of no price changing during the liquidity trap is made in the context of a modern New Keynesian DSGE model. Indeed, the long-run properties will be unaffected as this because of the ergodicity properties of the Taylor and Calvo contract frameworks- remember I was able to demonstrate the former very easily in Subsubsection 3.1.2, the proof for Calvo will be given in SubSection 3.4. **skeletal Intuition** WHAT STANDS UP IMPROVEMENTS WEAKNESSES... ZLB models M/S models Simsek / Schliefer / Lorenzoni ... wealth of other explanations ... This has immediate implications for the recent class of liquidity trap models such as which use Calvo pricing and assume a deflationary steady state with Markov reversion to a non-stochastic steady state. The non-stochastic steady state will never be attained

3.3 Stochastic Environment

Theorem 2. *If the distribution $F_T(\pi_T)$ is non-degenerate at zero (i.e. $Pr_T(\pi_T = 0) \neq 1$) $\forall T \geq 1$ then price dispersion $E\hat{\Delta}_T = \ln(\frac{\Delta_T}{\Delta_{SS}}) \forall T \geq 0$ is first order even when evaluated relative to the non-stochastic steady state $\Delta_{SS} = 1$*

Proof. $Pr(\pi_T = 0) \neq 1$ then $Pr(p_T^* \neq P_T) > 0$ from proposition 1 this implies $Pr(\Delta_T > 1) > 0$. This implies $Pr(\hat{\Delta}_T > 0) > 0$, proposition 1 also tells us that $\hat{\Delta}_T \geq 0$ therefore we can invoke Chebyshev's inequality to prove $E\hat{\Delta}_T > 0$. \square

Proof. The result is trivial if $\hat{\Delta}_0 > 0$ so consider the alternative that $\hat{\Delta}_0 = 0$ and $\Delta_0 = 1$. The result follows quickly from establishing that when there is non-trivial price rigidity we can only have $\Delta_t = 1$ and $\Delta_{t+1} = 1$ if $\pi_{t+1} = 0$. From Proposition 1 we know that $\Delta = 1$ only if all prices in the economy are equal. Therefore in both periods the price level and all reset prices will be equal to a common price. Formally, $P_t = p_{t-a}^* = \bar{p}_t$ and $P_{t+1} = p_{t+1-a}^* = \bar{p}_{t+1}$ for all ages of price a . Now with non-trivial price rigidity there is some reset price p_{t-j}^* for $j \geq 0$ which belongs to both price levels. Hence the two are equal and $\pi_{t+1} = 0$. Therefore for $\pi_T \neq 0$ implies $\Delta_{T+1} > 1$ and $\hat{\Delta}_{T+1} > 0$ for $T \geq 1$. Hence if $Pr(p_T^* \neq P_T) > 0$ then

$$Pr(\hat{\Delta}_T > 0) > 0$$

and we know from proposition 1 that $\hat{\Delta}_T \geq 0$ therefore $E\hat{\Delta}_T > 0$ follows from a Chebyshev's inequality argument which I produced in the proof of proposition 1. \square

The precise argument is the same as the one used to prove the first part of proposition 1 and is detailed in full in the appendix.

The reason for this result is an error in the microfoundations of the New Keynesian model. Essentially, you cannot have a New Keynesian model without dispersion. The critical observation is that the limiting case $\alpha = 0$ belongs to the class of New Classical model such as Lucas [1972] Lucas [1973] Sargent and Wallace [1976]

3.4 A New Market Failure: Price Dispersion and the Firm

Just Intuition Currently

Proof. From proposition 1 we know that $E\Delta > 1$ \square

Remark 3. *Representing the economy by a linear perturbation around the non-stochastic steady state where $\Delta_t = 1$. Is equivalent to positing the existence of a representative firm.*

As we have seen in subsection 2.4 $\Delta = 1$ its derivative $\hat{\Delta} = 0$. Hence $\Delta = 1$ in the subspace in which we are taking the approximation. From theorem 1 this implies all firms set the same price and produce the same output. Therefore, suitably scaled up to the size of the economy any firm could function as a representative firm.

Theorem 3. *When $\Delta > 1$ a representative firm does not exist.*

Proof. By definition the candidate representative firm produces aggregate output Y . From the production function it must demand labor $L = \frac{Y}{A}$. This must equal the aggregate of the net output vectors $(y_i, -l_i)$ of individual firms, whose behavior is specified by all the equilibrium condition set out in Section 2.1-2.2. This means that:

$$Y = AL = \Delta Y$$

which yields a contradiction when $\Delta > 1$. \square

1 The significance of this result is that it prevents us invoking the first fundamental theorem of welfare economics, as we can to prove the Pareto efficiency of the equilibrium in the benchmark RBC model. Indeed, as we usually work with models that yield unique solutions to both the social planner and market equilibrium problems, this result prevents us designing optimal tax schemes to decentralize social planner solutions. In this case even if we could put in place a tax structure to correct the static distortions implied by imperfect competition as described in we would still have market failure. Price dispersion creates a *Keynesian inefficiency* that cannot be corrected by any non-state contingent tax system specified in advance of market trade because even unforecastable shocks

that change the optimal reset price cause inefficiency^{47,48}. This result encompasses and explains Alves [2014] who uses positive trend inflation to create price dispersion and notes the Pareto inefficiency ('lack of divine coincidence') and Damjanovic and Nolan [2010] who show that trend inflation acts like a negative productivity shock when added to a benchmark New Keynesian Phillips Curve like the one in Section 2.2. The explanation is that the price dispersion created by trend inflation is creating a wedge between the efficient representative firm of the zero inflation steady state Phillips curve and the inefficient aggregate output associated with price dispersion.

The approximation is invalid. Intuitively, you cannot approximate the behavior of an economy characterized by a certain type of market failure by studying its behavior around an improbable limiting case where this market failure ceases to exist. In a model where price dispersion is not allowed, constraints on price-setting cannot bind.

3.5 Lucas Critique and the New Keynesian Model

In 1976 Robert Lucas published his eponymous critique[?] which showed that an empirical relation representing a trade-off of interest to a policymaker might break down when the objectives or instruments available to that policymaker changed⁴⁹. The result is widely seen as the seminal paper in modern macroeconomics as commented by numerous interviewees in Snowdon and Vane [2005]. It was instrumental to the award of the 1995 Nobel Prize Lucas [1996].

He illustrated this concept with two applications to fiscal policy demonstrating how econometric relationships derived under one policy regime would break down under another. This finding was used to correctly predict that the marginal propensity to consume out of a temporary tax rebate would be significantly lower than indicated by a regression of consumption on disposable income where changes in *permanent income* provided a substantial portion of the observed variation in disposable income see Blinder [1981]Lusardi [1996]Taylor [2009]Auerbach and Gale [2009]Taylor [2011]Parker et al. [2013]Kaplan and Violante [2014] among a vast literature. With the Phillips curve (the focus here) he shows that the slope of a simple New Classical Phillips curve will flatten as monetary policy becomes more active.⁵⁰ This point gained popular traction as it offered an explanation for the breakdown of the Old Keynesian Phillips curve relation between inflation and unemployment Blackburn and Ravn [1992] Ravn and Sola [1995] Benati [2006].⁵¹ Formally, a statistical relationship is

⁴⁷Correia et al. [2008] avoid this inefficiency by making the price level non-stochastic, which means the optimal reset price will never change in response to news.

⁴⁸In fact, markets meet continuously but tax changes operate at a considerable lag. In this case the inefficiency result here retains trivially see Section 8 ??? In reality governments do not design tax policies with a view to correcting the effects of imperfect competition this is usually delegated to competition authorities with more modest aims who use price regulation and other measures rather than taxation.

⁴⁹The Lucas critique is well understood in policy circles where it goes by the name Goodhart's law after policy adviser Charles Goodhart's adage that: "Any observed statistical regularity will tend to collapse once pressure is placed upon it for policy purposes." Goodhart [1984]

⁵⁰This part of the paper is actually less original it is based on his earlier paper Lucas [1973] and is implicit in all formulations of the natural rate hypothesis whether or not rational expectations are imposed.

⁵¹Whether Lucas' explanation was correct remains a point of contention in the literature see Blinder and Newton [1981]Taylor [1999]Ireland [1999]Orphanides [2003]Lubik and Schorfheide

said to pass the Lucas critique if its parameter set \mathfrak{B} is independent of the parameters of the set of policy parameters \mathfrak{P} . This is an incredibly restrictive criterion. It requires complete monetary neutrality a property which no monetary model Keynesian or Classical can meet.⁵² For example in the RBC framework both money-in-the-utility function and cash-in-advance constraints induce non-neutrality in response to unexpected monetary disturbances. This causes a dependence between the parameters of the statistical relationship between monetary and real variables and the structural parameters of the monetary policy process- determining the magnitude of the monetary disturbances Sidrauski [1967] Clower [1967] Lucas and Stokey [1987] Cooley and Hansen [1989]⁵³. In New Monetarist models money is not super-neutral because the trend rate of inflation can directly affect the technology of exchange Williamson et al. [2010]. In the New Keynesian tradition it is known that the trend rate of inflation effects both the business cycle dynamics and the long-run equilibrium Ascari and Sbordone [2014].⁵⁴ In fact I am able to extend this result to derive a mapping between the parameters of monetary policy reflecting the Central Banks policy preferences and the reduced form relationships among all variables- a result not previously possible because of the 'Divine Coincidence' discussed in Section 5. Lucas was aware of the stricture of his critique. In the paper's conclusion he suggests two approaches that have proved fruitful. The first is to effectively circumvent the critique by arguing the economy has a vector of hyper-parameters *theta* that are invariant to policy. This simplest way to be operationalize this would be through a random coefficient model where $\theta_t = \bar{\theta} + \epsilon_t$ with ϵ_t distributed independently from the regressors and the true shocks to the model as well as having a bounded first moment- the model can be estimated consistently- Snijders and Bosker [2011].⁵⁵ In a similar spirit are models with stochastic regime-switching

[2004] Coibion and Gorodnichenko [2011] Blinder and Rudd [2012] Needham [2015] for various evidence and perspectives.

⁵²The only model that would satisfy this condition with respect to the parameters of the monetary policy is a pure real business cycle model with no money holding for its own sake, perfect information and full flexibility in prices, wages and money holding. However, this neutrality result is sufficiently trivial that such models are never used for monetary policy analysis and the money sector is always left out take for example Kydland and Prescott [1982] Long Jr and Plosser [1983] Plosser [1989] King and Rebelo [1999].

⁵³Consult Walsh [2010] for a textbook exposition of these models.

⁵⁴These authors follow the convention of using the non-stochastic steady state as their equilibrium concept- which I show to not be the best way to characterize a dynamic stochastic equilibrium. However, the results would carry over naturally to the central moments of the ergodic distribution- using the arguments derived in Stokey [1989]???

⁵⁵Time-varying parameter models remain popular in the forecasting literature where out-of-sample prediction tests and econometric devices such as principal component and shrinkage priors are now used to reduce much of the over-fitting common in Lucas' time when data sets were shorter and computational arsenals weaker see Meese and Rogoff [1983] Tashman [2000] Stock and Watson [2002] Bernanke and Boivin [2003] Korobilis [2013] Koop and Korobilis [2013] Karlsson [2013] Giannone et al. [2015]. They have recently been applied to policy analysis often in the context of full DSGE models see for example Stock and Watson [2002] Cogley and Sargent [2005] Primiceri [2005] Boivin [2006] Ireland [2007] Perron and Wada [2009] Korobilis [2013]. The limitation with these models is that only a very restricted set of the parameters are allowed to vary or strong priors are imposed. This becomes particularly problematic with large models such as Smets and Wouters [2003] Smets and Wouters [2007] Schmitt-Grohe et al. [2007] Christiano et al. [2005] including recent models emphasizing financial frictions like Christiano et al. [2014] Del Negro et al. [2015] where the availability of financial data restricts the estimation period and makes incorporating parameter heterogeneity impractical. Whereas, with the parsimonious model I derive here it should be much easier to incorporate heterogeneity in the basic structural parameters without overfitting.

discussed earlier. The near ubiquitous practice of filtering also helps remove the effect from unsystematic policy change (EXAMPLE HERE?). The second approach is quantitative policy simulation- where a dynamic stochastic general equilibrium model is simulated under different policy parameters- see Kydland and Prescott [1996] for an exposition. The strong assumptions that agents know the true structural parameters has been relaxed in an extensive learning literature.⁵⁶

Since all useful models fail the Lucas critique it is necessary to develop a new criterion to officiate which econometric relationships are "useful for policy" in the sense of being *representative* of the *structural model* of the economy and consistent with the two agreed types of good practice. To do so I define some terms two economic and four mathematical. The first is an structural model. A structural model is a system of relationships representing the behavior of the economy and the response of agents to well-defined incentives. I have in mind DSGE models although the definition is fully general comprising non-stochastic as well as partial equilibrium models, instances of multiple equilibrium and even systems that do not possess well-defined equilibrium at all.⁵⁷

Definition 9. A structural model \mathcal{S} consists of two vector-valued time series processes- the endogenous variables $\{Z_t\}$ and errors $\{u_t\}$ defined respectively on state spaces \mathcal{Z} and \mathcal{U} along with a parameter vector $\gamma \in \Gamma$. The path of the dynamical system is determined by the continuous function $F : \Gamma \times \mathcal{Z} \times \mathcal{U} \rightarrow \Gamma \times \mathcal{Z}$ such that $F = (I_\Gamma, f)$ where I_Γ is the identity map on Γ and $E_t Z_{t+1} = f(\gamma, Z_t, u_t)$.

The focus on continuity is to admit the application of topological principles. Be aware that continuity on a metric space is a weaker notion than $\epsilon - \delta$ continuity familiar from introductory real analysis courses. Recall that a *continuous function* has the property that the inverse image of every open set is open⁵⁸. This is an axiom in point-set topology. Readers may want to consult Ok [2007] for a proof that this holds for the family of open-intervals under continuous mappings between Euclidean spaces. The σ algebra (the family of measurable sets) of a metric space can include (a countable subset of) discrete mass points as well as open intervals as with the familiar Lebesgue measure. For example any measurable function between discrete metric spaces is continuous because the topology is discrete. In this case since the identity map is continuous and the Cartesian product of continuous functions is continuous if and only if each function is continuous. Continuity of F simply requires that f be continuous in u_t for all $(Z_t, \gamma) \in \mathcal{Z} \times \Gamma$ with respect to the topology \mathcal{U} and

⁵⁶see for example Marcet and Sargent [1989] Bullard and Suda [2016] Evans and Honkapohja [2003] Gaspar et al. [2006] Milani [2008] Bianchi [2013] Matthes et al. [2015] Cogley et al. [2015]

⁵⁷Limitations of space prevent me from adequately reviewing these literatures here. Azariadis and Stachurski [2005] reviews early application of multiple equilibrium models in a growth context Brito and Venditti [2010] Antoci et al. [2011] Kikuchi and Stachurski [2009] are more recent. Business cycle models of non-linear dynamics include Masson [1999] Benhabib et al. [2002a] Benhabib et al. [2002b] Kaas and Madden [2005] Benhabib and Eusepi [2005] Guegan [2009] Brito et al. [2013] many admit chaotic dynamics which mean that the economy will never reach equilibrium. Limitations of chaotic modeling are discussed later. Most of these models do not possess the equilibrium concept (ergodic distribution) that is the focus of analysis here.

⁵⁸For example the identity map is continuous because the inverse image of every open set is itself.

Z_t for all $(\gamma, \mathcal{U}_t) \in \Gamma \times \mathcal{U}$ and so on. Consult Waldmann [2014] for proofs. In an analogous fashion I define an *econometric* model

Definition 10. An econometric model \mathcal{E} is comprised of two vector-valued processes- endogenous variables $\{V_t\}$ and shock process e_t with state spaces \mathcal{V} and \mathcal{E} respectively along with identified parameter vector $\theta \in \Theta$. The path of the dynamical system is determined by the continuous function $G : \Theta \times \mathcal{V} \times \mathcal{E} \rightarrow \Theta \times \mathcal{V}$ where $G = (I_\Theta, g)$ and $E_t V_{t+1} = g(\theta, V_t, e_t)$ and the transition function

It admits calibrated as well as estimated models. The requirement for identification (as defined formally in Section 2) corresponds to the possibility for conducting hypothesis tests about $\theta \in \Theta$ using statistics of $\{V_t\}$. Here V_t are observed from aggregate data or calculated from aggregate observed variables as stipulated by the econometric model. For example, the three equation New Keynesian model of Section 2 is an econometric model in which it is calculated from the observed variables (π_t, y_t) that $\hat{\Delta}_t = 0 \forall t$ by log-linearization about the non-stochastic steady state so $V_t = (\pi_t, y_t, \hat{\Delta}_t) = (\pi_t, y_t, 0)$.

The first two mathematical terms relate to vector spaces and the second pair come from topology. Consider a vector $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$ a subvector \mathbf{x}^s whose elements are a subset of those of \mathbf{x} formally $\mathbf{x}^s = (\mathbf{x}_{j_1}, \dots, \mathbf{x}_{j_k})$ where $\{j_1, \dots, j_k\} \subset \{1, \dots, n\}$

A *homeomorphism* is a function $h : X \rightarrow Y$ between two topological spaces⁵⁹ (X, T_X) and (Y, T_Y) with the following properties

- i h is a bijection⁶⁰
- ii h is continuous
- iii h^{-1} is continuous

Two functions $g_1 : X \rightarrow X$ and $g_2 : Y \rightarrow Y$ are *topologically conjugate* if there exists a homeomorphism such that $g_1 \circ h = h \circ g_2$ where \circ indicates a composite function so $g_1 \circ h = g_1(h(x))$.

Topology is the branch of mathematics that deals with shape and the relations between neighborhoods of points. Homeomorphism is the equivalence relation in topology so two spaces are *topologically equivalent* if and only if a homeomorphism can be found between them. Two functions are topologically conjugate if there is a homeomorphism between their images. Topology is an extension of Euclidean geometry. If a topological space is a geometric object then a homeomorphism is a continuous stretching or bending of that object with gluing and cutting forbidden. For two topological spaces to be homeomorphic one must be able to map from each to the other without mapping to the same point twice.

⁵⁹Recall that a *topological space* is a collection of open sets containing all countable union and finite intersection of these open sets. An open set is an abstract concept in topology so any set fulfilling these properties can be defined as open. Wherever convenient the conventional open intervals in Euclidean space will be used as open sets. This is the so called *usual topology* on \mathbb{R}^n .

⁶⁰Recall that a bijection is a correspondence that is *injective* (one-to-one) so

$$\forall a, b \in X \ h(a) = h(b) \Rightarrow a = b$$

and *surjective* (onto)

$$\forall y \in Y \ \exists x \in X, \ h(x) = y$$

so each element of Y has a corresponding element of X .

Topological conjugacy has been used extensively in the study of dynamical systems to explain why it is legitimate to approximate a non-linear systems by a linear approximation local to a hyperbolic fixed point⁶¹. The seminal result the Grobman-Hartman theorem Grobman [1959]Hartman [1960] and its extensions theorem will be discussed in greater detail later on in this section. My argument is that topological conjugacy can be used to precisely specify what it means for an econometric model to *pass* the Lucas critique with respect to the arguments set out in Lucas Jr [1976]. I shall refer to this condition here as the *Extended Lucas critique*.⁶²

Definition 11. An econometric model \mathcal{E} governed by the function F passes the *Extended Lucas critique* with respect to the ambient an structural model \mathcal{S} governed by the function G if F and G are topologically conjugate.

with parametrization family Γ and shock set u_T^S if it fulfills two conditions:
Parameter Mapping There exists a subset $\Gamma^0 \subset \Gamma$ with mapping F where $F : \Gamma^0 \rightarrow \Lambda$
Topological Conjugacy There exists a homeomorphism between the predicted values of the endogenous variables Z_{t+1} (corresponding to $u_t^S = \bar{u}^S$) given their current realization z_t under the structural model \mathcal{S} and their corresponding prediction under the econometric model \mathcal{E} (corresponding to $u_t^E = \bar{u}^E$) for all parametrization $\gamma \in \Gamma$. A homeomorphism is a continuous function with a continuous inverse. Let Z_t It follows immediately that any equation of a DSGE model passes this critique with respect to the DSGE itself through the identity map on $\Gamma^0 = \Lambda$.

3.6 Solving for a Phillips Curve

Appendix

A Results from Subsection 2.6

This section presents a more details of the derivations related to the persistence problem with the benchmark New Keynesian model, alongside several extensions which indicate the robustness of the puzzle.

A.1 Eigenvalues and Convergence

To begin with the expression for the eigenvalues of the matrix A is as follows.

$$\lambda_1 = \frac{\sigma^{-1}(a_y + \omega\beta^{-1}) + 1 + \beta^{-1} - \sqrt{[\sigma^{-1}(a_y + \omega\beta^{-1}) + 1 + \beta^{-1}]^2 - 4\beta^{-1}(1 + \sigma^{-1}(a_y + \omega a_\pi))}}{2}$$

$$\lambda_2 = \frac{\sigma^{-1}(a_y + \omega\beta^{-1}) + 1 + \beta^{-1} + \sqrt{[\sigma^{-1}(a_y + \omega\beta^{-1}) + 1 + \beta^{-1}]^2 - 4\beta^{-1}(1 + \sigma^{-1}(a_y + \omega a_\pi))}}{2}$$

⁶¹Recall that a hyperbolic fixed point is an equilibrium of a dynamical system with no center manifold. For a differentiable system a fixed point is hyperbolic if and only if its Jacobian (evaluated at the fixed point) has no eigenvalues with zero real part in continuous time and the unit circle for the analogous discrete time matrix. The discrete time case will be covered in more detail later in this section. For more detail, extensions and exposition readers should consult Arnold [2013].

⁶²Acemoglu [2008], de la Fuente [2000], Stachurski et al. [2009]

Note that both are positive and the larger eigenvalue λ_2 is always greater than one. We need both to be outside the unit circle for unconditional convergence. In the case where the discriminant term under the square root is negative the solution takes the form $x_t = e^{-\gamma t}(A\cos(zt) + B\sin(zt))$ which will converge non-monotonically when the real part of both eigenvalues is outside the unit circle ($\gamma > 0$) see for example Simon and Blume [1994] for an exposition of this case. The econometric exercise in the following sub-section indicates that the conditions for determinacy and real roots are both met for the majority of calibrations and when the model is estimated for US data.

A.2 Full Solution for Benchmark Model

The coefficients for inflation are

$$\zeta_\pi^1 = \frac{-\sigma\beta\omega^2(\lambda_2^{-(1+i)} - \lambda_1^{-(1+i)})}{\sigma\beta^2(\lambda_2 - \lambda_1)}$$

$$\zeta_\pi^2 = \frac{\beta\omega^2(\lambda_2^{-(1+i)} - \lambda_1^{-(1+i)})}{\sigma\beta^2(\lambda_2 - \lambda_1)}$$

$$\zeta_\pi^3 = \frac{\sigma\omega[(\beta\lambda_2 - 1)\lambda_1^{-(1+i)} - (\beta\lambda_1 - 1)\lambda_2^{-(1+i)}]}{\sigma\beta^2(\lambda_2 - \lambda_1)}$$

For output we have

$$\zeta_y^1 = \frac{\sigma\beta\omega[(\beta\lambda_2 - 1)\lambda_2^{-(1+i)} - (\beta\lambda_1 - 1)\lambda_1^{-(1+i)}]}{\sigma\beta^2(\lambda_2 - \lambda_1)}$$

$$\zeta_y^2 = \frac{-\beta\omega[(\beta\lambda_2 - 1)\lambda_2^{-(1+i)} - (\beta\lambda_1 - 1)\lambda_1^{-(1+i)}]}{\sigma\beta^2(\lambda_2 - \lambda_1)}$$

$$\zeta_y^3 = \frac{\sigma(\beta\lambda_1 - 1)(\beta\lambda_2 - 1)(\lambda_2^{-(1+i)} - \lambda_1^{-(1+i)}) - \omega[(\beta\lambda_2 - 1)\lambda_2^{-(1+i)} - (\beta\lambda_1 - 1)\lambda_1^{-(1+i)}]}{\sigma\beta^2(\lambda_2 - \lambda_1)}$$

Finally for interest rates

$$\zeta_i^1 = \frac{-a_\pi\sigma\beta\omega^2(\lambda_2^{-(1+i)} - \lambda_1^{-(1+i)}) + a_y\sigma\beta\omega[(\beta\lambda_2 - 1)\lambda_2^{-(1+i)} - (\beta\lambda_1 - 1)\lambda_1^{-(1+i)}]}{\sigma\beta^2(\lambda_2 - \lambda_1)}$$

$$\zeta_i^2 = \frac{a_\pi\beta\omega^2(\lambda_2^{-(1+i)} - \lambda_1^{-(1+i)}) - a_y\beta\omega[(\beta\lambda_2 - 1)\lambda_1^{-(1+i)} - (\beta\lambda_1 - 1)\lambda_2^{-(1+i)}]}{\sigma\beta^2(\lambda_2 - \lambda_1)}$$

$$\zeta_i^3 = \frac{a_\pi\beta\omega[(\beta\lambda_2 - 1)\lambda_1^{-(1+i)} - (\beta\lambda_1 - 1)\lambda_2^{-(1+i)}] + a_\pi\omega^2(\lambda_2^{-(1+i)} - \lambda_1^{-(1+i)}) + a_y\sigma(\beta\lambda_1 - 1)(\beta\lambda_2 - 1)(\lambda_2^{-(1+i)} - \lambda_1^{-(1+i)}) - a_y\omega(\beta\lambda_2 - 1)\lambda_2^{-(1+i)} - (\beta\lambda_1 - 1)\lambda_1^{-(1+i)}}{\sigma\beta^2(\lambda_2 - \lambda_1)}$$

None of the three interest rate coefficients can be decisively signed which explains the counter-intuitive or uncertain signs of all other coefficients. This confirms the interpretation in the text that the error terms do not correspond with our intuition of what a shock is when policy responds to contemporaneous variables. For example, $\zeta_\pi^1 < 0$ implies that a preference shock which causes

the household to move consumption from the next period to the present actually causes present consumption to *fall*. This is a consequence of the increase in real interest rate associated with the Taylor principle. When there are repeat eigenvalues the form of the solution is different. The repeat eigenvalue is $\lambda = \sigma^{-1}(a_y + \omega\beta^{-1}) + 1 + \beta^{-1}$ as many terms are zero in the expansion we find that see Halmos [1958]

$$A^{-(1+i)} = -i\lambda^{-(1+i)} + (1+i)\lambda^{-i}A^{-1}$$

. General solution equation (37) then allows us to calculate the *zeta* coefficients.

A.3 Persistence with Forward-Looking Policy

The lack of persistence finding generalizes naturally to a class of forward-looking policy stance, which includes the infinite horizon loss functions studied in optimal policy Sections 5 and 6 but also allows for differing weights on output and inflation objectives, policy horizons and discounting behavior. I require the following restrictions on every admissible loss function $L \in \mathcal{L}$ the family of admissible loss functions.

Assumption 3. *All moons are made of cheese!!!*

A.4 Persistence in Generalized Taylor

A.5 Persistence with Capital Goods

B Proofs from Section 3

This item proves the three propositions about price dispersion in the stochastic New Keynesian model mentioned in Sections 2 and 3.

B.1 Proof of Lemma 2

Proof. The proof that $\Delta \geq 1$ is an application of Jensen's inequality. First define two functions.

$$g(p_i) = p_i * \left(\frac{p_i}{P}\right)^{-\theta}$$

$$\phi(p_i) = \left(\frac{p_i}{P}\right)^{\frac{\theta}{\theta-1}}$$

We need to assign a probability measure for the prices. Any non-singular measure assigning zero probability $p_i = 0$ at every history will suffice. Note that the construction of the price level ensures that then $P > 0$ with probability one. Therefore, we know that ϕ is strictly convex on every measurable set since $\frac{d^2\phi}{dp_i^2} = \frac{\theta}{(\theta-1)^2} \frac{\phi(p_i)}{p_i^2} > 0, \forall p_i > 0$. Although in the first part of the proof I will only use the weak convexity property.

Note that $P = \int_{\Omega} g \, d\mu$. Now since ϕ is a convex function defined on a metric space it follows from theorem 7.12 (p. 265) in Aliprantis and Kim that it has a sub-derivative at every point. Hence we may select exists a and b such that:

$$ap^* + b \leq \phi(p^*)$$

For all possible reset prices p^* and for the particular value $p^* = P$

$$aP + b = \phi(P)$$

It follows that:

$$\phi \circ g(p^*) \geq ag(p^*) + b$$

For all p^* since we have a probability measure the integral is monotone with $\mu(\Omega) = 1$. Note that:

$$\begin{aligned} \Delta &= \int_{\Omega} \phi \circ g \, d\mu \\ &\geq \int_{\Omega} (ag + b) \, d\mu \\ &= a \int_{\Omega} g \, d\mu + \int_{\Omega} b \, d\mu \\ &= aP + b \\ &= \phi(P) \\ &= 1 \end{aligned}$$

□

Where I have used respectively the monotonicity of the Lebesgue integral, the linearity of the Lebesgue integral and the definition of the functions see Royden and Fitzpatrick (p80-82).

For the second part we need to be clear about the nature of the measure used. It is a discrete measure corresponding to price dispersion statistics defined at each time t . As a probability measure it is defined by the triplet (Ω, Σ, μ) where Ω is a probability space Σ is a sigma-algebra of sets and μ is a probability measure defined for every set in Σ . Ω_t is the set of all prices in the economy at time t . Σ_t is the set of all subsets (or power set) of Ω_t denoted $\Sigma_t = \mathcal{P}(\Omega_t)$ and μ_t is the share of particular prices in the economy at time t . To generalize the result across models simply modify the probability measure. The definition of Σ_t will be the same for all discrete time models but Ω_t and μ_t will change. For the Calvo model without indexation used here they are as follows:

$$\begin{aligned} \Omega_t &= \{\dots, p_{-1}^*, p_0^*, p_1^*, \dots, p_t^*\} \\ \mu_t(p) &= \sum_{-\infty}^t \delta_T(p) \alpha^{t-T} (1 - \alpha) \end{aligned}$$

Where p_T^* indexes the reset price at time t and δ_T is the indicator function for time T . Defined as $\delta_T(p) = \begin{cases} 1 & \text{if } p = p_T^* \\ 0 & \text{otherwise.} \end{cases} \quad \forall 0 \leq T \leq t$ Note that the

incongruous feature of having an infinite history of reset prices is not necessary to prove Lemma 1 for the Calvo model- the result would pass with a finite history of reset prices starting at p_0^* with the measure $\mu_t(p) = \sum_{T=1}^t \delta_T(p) \alpha^{t-T} (1 - \alpha) + \delta_0(p) \alpha^t$. However, the proof of the existence of the stochastic steady state in Theorem ??? does rely on an infinite history i.e. allowing the limit $t - T \rightarrow \infty$. As do subsequent generalizations.

For the Yun [1996] model where the Calvo pricing firms not allowed to re-optimize index to trend inflation $\bar{\pi}$ they are:

$$\Omega_t = \{\dots, (1 + \bar{\pi})^{t+1} p_{-1}^*, (1 + \bar{\pi})^t p_0^*, (1 + \bar{\pi})^{t-1} p_1^*, \dots, p_t^*\}$$

$$\mu_t(p) = \Sigma_{-\infty}^t \hat{\delta}_{t,T}(p) \alpha^{t-T} (1 - \alpha)$$

Where $\hat{\delta}_{t,T}(p) = \begin{cases} 1 & \text{if } p = (1 + \bar{\pi})^{t-T} p_T^* \quad \forall 0 \leq T \leq t \\ 0 & \text{otherwise.} \end{cases}$

For the Generalized Taylor Economy:

$$\Omega_t = \{\dots, p_{t-(J-1),J}^*, p_{t-(J-2),J}^*, p_{t-(J-2),J-1}^*, \dots, p_{t-1,2}^*, p_{t-1,3}^*, \dots, p_{t-1,J}^*, p_{t,1}^*, p_{t,2}^*, \dots, p_{t,J}^*\}$$

$$\mu_t(p) = \sum_{j=1}^J \sum_{k=0}^{j-1} \delta_{t-k,j} \gamma_j / j$$

Note that the second subscript now indicates contract length and J is the maximum contract length. γ_j is the share of firms with contract length j . There is staggered pricing for each contract length so $1/j$ is the fraction of firms with contract length j resetting at a given time. The need to generate staggered nominal adjustment within each sector is the reason why we cannot begin the contract history at the date $t = 0$. Note that unlike with Calvo there is not a single reset price in each period but one for each contract length 1 through j . Finally, other models of state dependent pricing can be admitted trivially by making parameters in the measure dependent on the parameters and history of shocks.

To derive the conditions for $\Delta > 1$ we need to employ the strict convexity property of ϕ which is as follows:

$$\phi(sp + (1 - s)p') < s\phi(p) + (1 - s)\phi(p')$$

for any s in $(0,1)$ and $p \neq p'$. Select any p' we know from its weak convexity property that ϕ has a sub-derivative at p' so:

$$ap + b \leq \phi(p)$$

$$ap' + b = \phi(p')$$

Now here begins a contradiction argument. Suppose another point p'' had the same sub-derivative then:

$$ap'' + b = \phi(p'')$$

Now consider a convex combination $p''' = sp' + (1 - s)p''$. As ϕ is strictly convex we know that:

$$s\phi(p') + (1 - s)\phi(p'') > \phi(p''')$$

However, as we have assumed they have the same subderivative we obtain a contradiction:

$$s(ap' + b) + (1 - s)(ap'' + b) = ap''' + b > \phi(p''')$$

Therefore for all $p^* \neq P$:

$$\phi \circ g(p^*) > ag(p^*) + b$$

Now partition the sample space as follows: $\Omega_t^1 = \Omega_t \setminus \{p_T^* : p_T^* = P_t\}$ and $\Omega_t^2 = \Omega_t \setminus \Omega_t^1$. This allows me to decompose the condition for $\Delta = 1$ as follows:

$$\int_{\Omega_t^1} \phi \circ g \, d\mu + \int_{\Omega_t^2} \phi \circ g \, d\mu = \int_{\Omega_t^1} (ag + b) \, d\mu + \int_{\Omega_t^2} (ag + b) \, d\mu$$

Now we can cancel the second expression on each side because we know they are always equal on every set in Ω_t^2 and we can invoke uniform continuity of the Lebesgue integral to extend this to the sigma-algebra. Rearranging and reparametrizing with the function $h = \phi \circ g - (ag + b)$ yields the condition:

$$\int_{\Omega_t^1} h \, d\mu = 0$$

Now the next step is to prove that the set Ω_t^1 has measure zero. Since $h \geq 0$ I can apply Chebyshev's inequality which states that for any $\epsilon > 0$:

$$\mu(\{h > 0\}) \leq \frac{1}{\epsilon} \int_{\Omega_t^1} h \, d\mu = 0$$

. Take the union over sequences $\epsilon_k \searrow 0$ to obtain $\mu(\{h > 0\}) = 0$. Now note that:

$$p \in \Omega_t^1 \Rightarrow h(p) > 0 \Rightarrow p \in \{h > 0\}$$

So $\Omega_t^1 \subseteq \{h > 0\}$ thus $\mu(\Omega_t^1) = 0$. Now this means every subset of Ω_t^1 must have zero probability including every individual reset price p however this contradicts the definition of Ω that a positive fraction of firms are selling at price p . Therefore Ω_t^1 is the empty set and $\Omega_t^2 = \Omega$, so $\Delta = 1$ if and only if every reset price $p^* = p$. To prove $p_i = P$ write the price level as $P = \int_i p_i (\frac{p_i}{P}) d\mu$. Note that with all firms setting the same price p_i is independent of μ which with a dispersed price level would reflect the share of firms resetting prices at a certain date. We can therefore factorize p_i from the integral to leave $P = p_i \Delta$ since I have already shown that $\Delta = 1$ when all $p_i = P$, the proof is complete.

B.2 Definition 1 and Lemma 3 Material

This subsection contains extensions of Lemma 3 to cover various nominal indexation schemes proposed in the literature and the example cited in the text.

B.2.1 Taylor Pricing Example

Here is an example where under Taylor contracts non-zero inflation eliminates price dispersion. All contracts last two periods so the price level and dispersion are given respectively by

$$P_t^{1-\theta} = \frac{1}{2}(p_t^*)^{1-\theta} + \frac{1}{2}(p_{t-1}^*)^{1-\theta}$$

$$\Delta_t = \frac{1}{2}\left(\frac{p_t^*}{P_t}\right)^{-\theta} + \frac{1}{2}\left(\frac{p_{t-1}^*}{P_t}\right)^{-\theta}$$

Suppose $\theta = 2$, $p_0^* = 1$, $p_1^* = 2$ which solves to give price level $P_1 = \frac{4}{3}$ and price dispersion $\Delta_1 = \frac{10}{9}$. Now consider time $t = 2$ the firms that set there

price in period 0 now get to reset their price. Therefore the reset price $p_0^* = 1$ is replaced by p_2^* with $p_1^* = 2$ the other price in the economy. Now applying Lemma 2 $\Delta_2 = 1$ if and only if $p_2^* = 2$. This implies $P_2 = 2$ then inflation is non-zero in fact $\pi_t = 50\%$.

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