Lemma 1.21

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Lemma 1.21. *If $x > 0$, then $x^{-1} > 0$*

Proof. Proof by contradiction. Case 1: Let $x > 0$ and suppose $x^{-1} < 0$. Then $x \cdot x^{-1} < x \cdot 0$ by axiom 1.16. Since $x \cdot x^{-1} = 1$ by axiom 1.11 and $x \cdot 0 = 0$ by axiom 1.17, then $1 < 0$. which is a contradiction. Case 2: Let $x > 0$ and suppose $x^{-1} = 0$. By Lemma 1.11, $x^{-1} \in \mathbb{R}$ and $x \cdot x^{-1}$=1. If $x^{-1} = 0$, then $x \cdot x^{-1}$=$x \cdot 0$=0 by Lemma 1.17. That is, 1=0. which is a contradiction. Thus $x^{-1} \neq 0$. Therefore, if $x > 0$, then $x^{-1} > 0$. □