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# Chapter 1

## Introduction to Simulation

### 1.1 Introduction

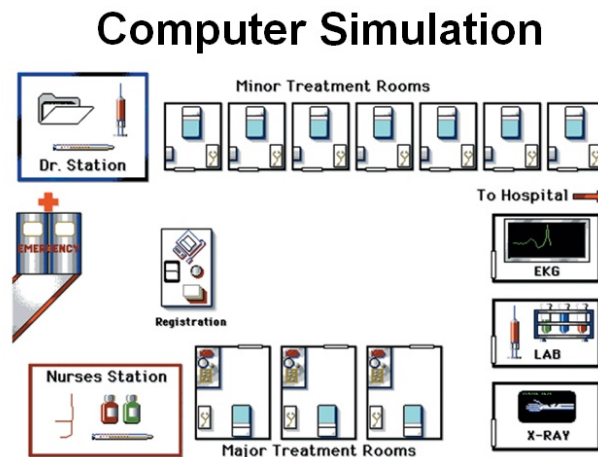


Figure 1.1: Computer Simulation (Heizer, 2014)

1. This chapter provides a general introduction to area of
  - modeling and
  - analysis of probabilistic modelsusing simulation.
2. Simulation methodology involves a set of techniques for
  - representing operational aspect and
  - relationships in a model by sampling variables and
  - using this to estimate one or more parameter of interest (Seila, *et al.*, 2003).

3. It is important to understand not only
  - the method and technique developing and
  - analysing a model using simulation,
 but also where the field of simulation fits into overall scheme of
  - operation research (OR) and
  - management science (MS) modeling.
4. When Bay Medical Center faced severe overcrowding at its outpatient clinic, it turned to computer simulation to try to reduce bottleneck and improve patients flow.
5. A simulation language called Micro Saint analysed current data relating to patient service times between clinic rooms.
6. By
  - simulating different numbers of doctors and staff,
  - simulating the use of another clinic for overflow, and
  - simulating a redesign of the existing clinic,
7. Bay Medical Center was able to make decision based on an understanding of both costs and benefits.
8. The resulted in better service at lower cost.

### 1.1.1 What is Simulation?

1. To imitate a real-world situation mathematically.
2. To study its properties and operating characteristics.
3. To draw conclusions and make action decisions based on the results of the simulation

## 1.2 Decision and Decision Models

1. Almost any time a decision is made, a model is used to aid the decision maker.
2. For example a behavioral model, " I believe that if I make this decision, the I will get the outcome" (Seila, *et al.*, 2003). This model, is an **implicit** and **ill-defined behavioral model** that involves **relationships and scenarios**.

3. Models can be **overt**( plainly or readily apparent) and **explicit**(stated clearly and in detail). The following are example of two models that is *explicitly* related to the decision variables.
  - A spreadsheet model that gives numerical relationships between decision variable and outcome of decision.
  - A linear programming model in which the decision maker's objective is explicitly related to the decision variables.
4. Decisions can take many forms.

**Example 1.2.1.**

- i) Business decisions to determine ware house capacities, production levels, staff hiring levels, or other parameters of a production system; selecting or rejecting certain investment opportunities; determining whether to increase the prime interest rate in a financial context; selecting a plan to market a new product; or determining how much money to retain for a self insurance plan.
  - ii) Engineering decisions whether to accept a design for a new automobile suspension, or how many processors to put in a newly designed computer system.
  - iii) Decision in statistics to estimate mean cost per year for college education in Malaysia using a random sample of college students in Malaysia. To make this decision, we need to use statistical model.
5. In operations research, we use explicit mathematical models to make decisions, so that any time a decision is made with the use of mathematical model, the work can be classified as operations research.
  6. *Management science* is a field involve the use of model to make **administrative** or **managerial decision**.
  7. These two fields are overlap, because most model used are mathematical nature. In practice , the title are used interchangeably as OR/MS or MS/OR.
  8. Operations research also referred to as quantitative analysis, management science or decision science.
    - a) System
      - i) A system is a set of interacting components or entities.
      - ii) The components operate together to achieve a common goal or objective.
      - iii) When the components work together, the system can perform tasks and accomplish objectives that individual components cannot.

**Example 1.2.2.**

Consider the system for stocking and selling goods at a grocery store.

- the arriving customers,
- the checkout counters and attendants,
- a customer service attendant,
- the queues or waiting lines for customers,
- the baskets for carrying groceries, and
- the store with its stock.

iv) Other systems?

- Hospital patients, rooms, personnel and equipment
- Air traffic control radar detectors, planes, runways and controllers
- Telecommunications with its messages, communication network and receiving stations
- Manufacturing machine centers, inventories, production schedules, and items produced

vi) Entities in a system may be physical entities or conceptual entities.

vii) Most systems in the real world are highly complex, so it is useful to divide the systems into subsystems.

viii) A subsystem is a part of a system that performs a specific task for the entire system.

**Example 1.2.3.**

A hospital is divided into subsystems emergency room, surgical suite, intensive care unit, maternity ward, medical wards, X-ray department ,laboratory, dietary, and laundry .

9. b) Model

- i) A model is an abstract and simplified representation of a system.
- ii) Models may be: iconic (look-alike the real entity), analog (explains a phenomenon by reference to some other occurrence) e.g. thermometer, schematic (drawing and pictures) e.g. map and mathematical (mathematical relationship).
- iii) Simulation model include all; iconic, analog, schematic and mathematical model. Example: Model of the checkout system
- iv) Models are prescriptive; they determine the optimal policy (eg. Linear Programming). Models are descriptive; they describe relationships and provide information for evaluation (eg. Queuing Model).
- v) Models may also be deterministic or probabilistic. Deterministic model all input variables are known and have constant values (not involved chance and

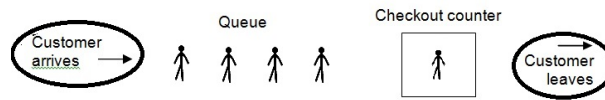


Figure 1.2: Checkout system

risk) (eg. EOQ model). Probabilistic (Stochastic) model at least one input variable is uncertain and with value that varies. Some data are described by probabilistic distributions (eg. Monte Carlo Simulation).

- vi) Models may be discrete or continuous. In mathematical programming, refers to the types of variables in the model. It may also refer to how model variables change over time.
- vii) Simulation models are descriptive, they simply evaluate measures of performance or the behavior of a system for a specific set of inputs.
- viii) Simulation models in this course are under probabilistic models. Some common examples of data that exhibit random behavior are consumer sales, machine operating times until failure, customer service times and project activity completion times.
- ix) Simulation models can be either discrete or continuous. For most systems, events occur at discrete points in time (customer arrivals, start times of production jobs, departures of ships from ports and so on. Simulating operations such as an oil refinery, we must incorporate variables such as temperature, pressure and material flow rates, these occur continuously over time. Simulation models in this course are under discrete simulations.
- x) Developing models to provide information for making decisions, the term decision model is frequently used. The model helps in making decisions by allowing us to understand how the system behaves and by predicting how the system will respond to various decisions. From various decisions the decision maker will choose the best.

#### c) Parameter

- i) A parameter is defined to be any numerical characteristics of a model or system. It is a number that describes something about the model.
- ii) There are two types of parameters:  
Input parameter -any parameter whose value is required as part of the model specification.

#### Example 1.2.4.

- The number of counters or servers.
- The mean inter-arrival time between customers.

- The service time distribution parameters.

Output parameter- any parameter whose value is determined by the system and its input parameters. Specifies some measure of the systems performance.

**Example 1.2.5.**

- The mean number of customers waiting in the system.
  - The mean time spent waiting in the queue.
  - The utilization of checkout counters.
- iii) The relationship between input parameters, the model and output parameters can be depicted as in the following diagram.

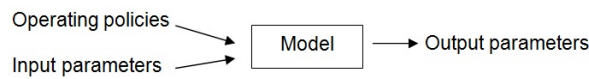


Figure 1.3: Relationship between inputs

- iv) Output parameters are the answers that we want the model to provide to aid us in decision making.
- v) We must specify how the model relates the output parameters to the input parameters. There are two basic ways to do this:
- Use mathematical analysis
  - Use simulation
- vi) A mathematical analysis will yield formulas and computational procedure to produce an exact value of the models performance.
- vii) A simulation, will yield a sample observations that can be used to compute a confidence interval for the performance measure, and therefore to estimate the value of the performance measure. Simulation cannot compute the exact value.



## 1.3 Definition of Simulation

1. Simulation is the process of building a model of a system or a decision problem and experimenting with the model to obtain insight into the systems behavior or to assist in solving the decision problem.
2. The two key elements of this definition are model and experiment.
3. To simulate means that to try to duplicate the features, appearance and characteristics of a real system.
4. The idea behind simulation is to imitate a real-world situation with a simulation model. The experiment using simulation model will not affect the real operations.
5. The term simulation model means a model that has been adapted to be analyzed with the use of simulation.
6. For situations in which a problem does not meet the assumptions required by standard analytical modeling approaches, simulation can be the best approach to modeling and solving problem.

## 1.4 Applications of Simulation

1. Analyzing the operations of:
  - Passenger flows at an airport terminal.
  - Patient care at an outpatient department.
  - Call center operations.
  - Product development.
  - Staffing in a manufacturing system.
  - Work flow in a financial services firm.
2. Application of simulation to business and industrial problems are increasing over the past 30 years, this is because to the three developments:
  - i) Fast and easy to use computers, available to most people employed.
  - ii) Efficient software is available.
  - iii) Number of students who study simulation has increased.

## 1.5 When Simulations Are Used

1. The early years (1950s-1960s)
  - Very expensive, specialized tool to use by large corporations (steel and aerospace).
  - Required big computers, special training.
  - Mostly in FORTRAN language.
  - Processing cost as high as \$600 to \$1000 per hour.
2. The formative years (1970s-early 1980s) Computers got faster, cheaper. Value of simulation more widely recognized, although most of the company quite large. Simulation was seldom considered until there was a disaster. Often used to clean up disasters in auto, aerospace industries.
  - Car plant; heavy demand for certain model.
  - Assembly line underperforming.
  - Simulated, problem identified.
  - But demand had dried up the result was too late. Simulation also found in academic area as a standard part of industrial engineering and operations research curriculum.
3. The recent past (late 1980s-1990s) Introduction to personal computer and animation. Wider acceptance across more areas
  - Traditional manufacturing applications
  - Services
  - Health care
  - Business processes Still mostly in large firms.
4. The present Began to mature during the 1990s. Many smaller firms accept the tool. Better animation, greater ease of use, faster computers, easy integration with other packages and the emergence of simulators have all helped simulation become a standard tool in many companies. Most managers admit that simulation can add value to their enterprise.

## 1.6 Types of Simulation Models

1. Gaming simulation involve the interaction of one or more persons with the simulation program (example: video games, war games, flight simulators and etc.).
2. Static simulation (Monte Carlo simulation) sampling observations and transforming them according to formulas or rules that compose the model. Repeated independently many times.
3. Dynamic system simulation observe the behavior of system models over time (example: inventory, queuing and etc.). Dynamic system simulation divide by two subareas, continuous simulation and discrete simulation. Continuous simulation state variables/attributes change continuously as time progresses (eg. Model of water flow in a river). Discrete simulations allow state variables/attributes (system quantities) to change only at discrete points in time. System quantities change are called events discrete event simulation (eg. Manufacturing systems that produce parts).

## 1.7 The Simulation Process

1. Develop a conceptual model of the system or problem under study. This step begin with understanding and defining the problem, identifying the goals and objectives of the study, determining the important input variables and defining output measures.
2. Build the simulation model. This includes developing appropriate formulas, equations, formulating the model design, collecting any necessary data and take into consideration the constraints involve, determining the probability distributions of uncertain variables and constructing a format for recording the results.
3. Verify and validate the model. Verification refers to the process of ensuring that the model is free from logical errors. Validation ensures that the model behaves the same as the real system or problem.
4. Design experiments using the model. We should design a complete set of experiments that we intend to conduct in order to improve the systems performance.
5. Perform the experiments and analyze the results. Run the appropriate simulations to obtain the information required to make an informed decision.

## 1.8 Advantages of simulation

1. It is relatively straightforward and flexible.
2. Software make simulation model very easy to develop.
3. It can be used to analyze large and complex real world situations that cannot be solved by conventional quantitative analysis models.
4. Simulation allows what-if? types of questions. This enables the decision maker evaluate / consider more than one alternative before choosing the best one.
5. Data for further analysis can easily be generated.
6. Avoids cost of real world experimentation.
7. Simulation do not interfere with the real-world system. With simulation, experiments are done with the model, not on the system itself.
8. Simulation allows us to study the interactive effect of several variables to determine which one is important.
9. The effects of changes in variables over many months / years can be obtained by computer simulation in a short time.
10. Serves as 'pre-service test'.

## 1.9 Disadvantages of simulation

1. Developing a good simulation model is often a long and complicated process especially for large and complex problems.
2. Simulation does not give us the optimal solution. It is a trial and error approach that produces different solutions in repeated runs.
3. The user must generate all of the conditions and constraints for solutions that they want to examine.
4. Each simulation model is unique. Its solutions and inferences are not usually transferable to other problems.

## 1.10 Problems

1. For the following systems, identify their subsystems, the input parameters and output parameters.
  - a) A restaurant.
  - b) A university.
  - c) A bank.
  - d) A supermarket.
  - e) A petrol station.
2. Simulation cannot compute the exact value. Mathematical analysis can compute the exact value. Give two examples of mathematical analysis.
3. Give three examples of simulation applications.

# Chapter 2

## Simulation on Spreadsheets

### 2.1 Static Simulation (Monte Carlo Simulation) on Spreadsheet

1. In Topic 1, we introduced the concept of a model as a useful representation for analyzing the possible consequences of decisions and the concept of a simulation as a sampling experiment concerning the outcomes of the model.
2. In this topic, we are going to develop the idea of simulation and to explore using static models.
3. Recall that, static model is one in which we do not record observations on the system over time.
4. The simulation consists of:
  - Generating random variates and combining them according to the formulas or rules of the model to produce an observation for the output parameter or performance measure.
  - Repeat this process a specific number of times or a specific number of replications in order to produce a collection of independent, identically distributed observations.
  - Then can be analyzed using standard statistical and graphical methods to show the distribution of the data and estimate parameters of this distribution, such as the mean performance measure.
5. Under static simulation the objective here is to demonstrate that a simulation can provide a reliable estimate of the desired output parameter if it is allowed to produce enough data.

## 2.1. STATIC SIMULATION (MONTE CARLO SIMULATION) ON SPREADSHEET13

6. Two other aspects of simulation are generation of random variates from some common distributions and analysis of output data. Five steps in Monte Carlo Simulation

STEP 1: Setting up a probability distribution for important variables.

STEP 2: Building a cumulative probability distribution for each variable in STEP 1.

STEP 3: Establish interval of random numbers for each variable.

STEP 4: Generating random numbers.

STEP 5: Simulating a series of trials.

### Example 2.1.1.

Harrys Auto Tire sells all types of tires, but Tire A accounts for a large portion of Harry overall sales. Harry wishes to simulate the daily demand for 10 days. Find the average daily demand.

STEP 1: Setting up a probability distribution for important variables. The daily demand for Tire A over the past 200 days is as follows:

Demand for Tire A	Frequency (days)	Probability
0	10	$\frac{10}{20} = 0.05$
1	20	0.1
2	40	0.2
3	60	0.3
4	40	0.2
5	30	0.15
Total	200	1

STEP 2: Building a cumulative probability distribution for each variable in STEP 1.

STEP 3: Establish interval of random numbers for each variable.

Demand for Tire A	Probability	Cumulative Probability	Random Number Interval
0	0.5	0.5	01 – 05
1	0.10	0.15	06 – 15
2	0.20	0.35	16 – 35
3	0.30	0.65	36 – 65
4	0.20	0.85	55 – 85
5	0.15	1.00	86 – 00

STEP 4: Generating random numbers.

- Random numbers may be generated for simulation problems in several ways such as using computers or by using random number table as follows.
- For this example, we can generate random numbers using the following random number table.

Partial table of random numbers (upper left corner)

52	06	50	88	53	30	10	47	99	37	66	91	35
37	63	28	02	74	35	24	03	29	60	74	85	90
82	57	68	28	05	94	03	11	27	79	90	87	92
69	02	36	49	71	99	32	10	75	21	95	90	94
98	94	90	36	06	78	23	67	89	85	29	21	25
96	52	62	87	49	56	49	23	78	71	72	90	57
33	69	27	21	11	60	95	89	68	48	17	89	34
50	33	50	95	13	44	34	62	63	39	55	29	30
88	32	18	50	62	57	34	56	62	31	15	40	90
90	30	36	24	60	82	51	74	30	35	36	85	01
50	48	61	18	85	23	08	54	17	12	80	69	24
27	88	21	62	69	64	48	31	12	73	02	68	00
45	14	46	32	13	49	66	62	74	41	86	98	92

Source: Reprinted from A Million Random Digits with 100000 Normal Deviates, Rand (New York: The Free Press, 1995).

STEP 5: Simulating a series of trials.

Simulate the demand for Tire A for 10 days.

Day	Random Number	Daily Demand
1	52	3
2	06	1
3	50	3
4	88	5
5	53	3
6	30	2
7	10	1
8	47	3
9	99	5
10	37	3
Total		29

Average daily demand =  $\frac{29}{10} = 2.9 \approx 3$  tires.

Use Excel: VLookup



## 2.2 Exercises

1. The AAA Health Insurance Company is concern with its cash outflows on a weekly basis. AAA is being considered for a large group policy. If AAA wins the contract and insures the group, the daily frequency of claims is estimated as follows:

Number of claims	Probability
0	0.05
1	0.06
2	0.12
3	0.34
4	0.25
5	0.11
6	0.07

Cost per claim (RM)	Probability
1000	0.30
1100	0.24
1200	0.22
1300	0.18
1400	0.06

- i) Calculate the expected number of claim per day and the expected cost per claim. Hence, estimate the weekly (5 working-day) cash outflow.
- ii) Simulate for 5 days to estimate weekly cash outflow. Compare your simulation result with (i) above. Are they the same? Explain.

Use the following random numbers for number of claim and cost per claim.

Number of claim: 44 06 61 93 59 42 39 87 34 63 88 54

Cost per claim : 35 44 57 99 66 47 43 52 31 72 15 68 80 87 98 20 16 80 43 62

2. Pantas Emergency Rescue Squad has gained reputation over the years from the public for its prompt and quality service. From previous experience, the company establishes the following probability distribution regarding receipt of number of emergency calls at night.

Number of calls	Probability
0	0.06
1	0.15
2	0.18
3	0.30
4	0.24
5	0.07

Pantas categorizes each call as either Minor, Normal or Major, and has found the following distribution for these categories:

Category	Probability
Minor	0.28
Normal	0.62
Major	0.1

Pantas sends a team comprising 3, 5 or 7 persons for Minor, Normal or Major calls respectively.

Use random numbers given below.

Number of Calls: 47 31 05 76 18 59 35 16 72 60

Type of Calls : 08 56 37 71 92 74 17 13 50 41 27 55 93 10 32 72 99 65 33 07 42 88  
22

- Simulate the emergency calls received by Pantas for 10 nights.
- Determine the average number of calls of each category per night.
- Determine the average number of calls received per night.
- Determine the average crew size required per night.

## 2.3 Static (Monte Carlo) Simulation (Cont)

### Example 2.3.1.

Daves Candies is a small family-owned business that offers gourmet chocolates and ice cream fountain service for special occasions such as Teachers Day, the store must place an order for special packaging several weeks in advance from their supplier. One product, Sweet Chocolate is bought for RM7.50 a box and sells for RM12. Any boxes that are not sold by May 16 are discounted by 50% and can always be sold easily. Historically Daves Candies has sold between 40 and 80 boxes each year with no apparent trend (either increasing or decreasing). If demand is exceed the purchase quantity, Dave loses profit opportunity. On the other hand, if too many boxes are purchased, he will lose money by discounting them below cost. Daves dilemma is deciding how many boxes to order for the Teachers Day customers.

**Solution** :

- If  $Q$  boxes are purchased and sales demand is  $D$ :

$$\text{Profit} = \begin{cases} 12D - 7.50Q + 6(Q - D), & \text{if } D \leq Q \\ 12Q - 7.50Q, & \text{if } D > Q \end{cases} \quad (2.1)$$

- Input to a simulation model of this situation would be:
  1. The order quantity,  $Q$  (decision variable)
  2. The various revenue and cost factors (constants)
  3. The demand,  $D$  (uncontrollable and probabilistic)
- The model output we seek is the net profit. If we know the demand, we can use equation (2.1)(above) or (2.1)(below) to compute the profit. Since demand is probabilistic, we need to be able to sample a value from the probability distribution of demand.
- For now, we simplify this problem by assuming that demand will be either 40, 50, 60, 70 or 80 boxes with equal probability ( $\frac{1}{5}$  or 0.2).
- Simulate this model for 10 replications with the number of order is either 40, 60 or 80 boxes. From the net profit (simulation results), determine which order quantity will be the best.

- Use the following random numbers:

Order	40	50	48	61	18	85	23	08	54	17	12
Order	60	27	88	21	62	69	64	48	31	12	73
Order	80	45	14	46	32	13	49	66	62	74	41

**Example 2.3.2.**

A large catalog merchandiser is planning to have a special furniture promotion a year from now. To do this, the company must place its order for the furniture now. It plans to sign a contract with the manufacturer for 3000 chairs at a cost of RM175 per unit, which the company plans to offer initially RM250 per unit. The promotion will last for eight weeks, after which all remaining units will be offered at half the initial price or RM125 per unit. The company believes that 2000 units will be sold during the first eight weeks. Determine the profit.

$P$  : the profit from the promotion

$C$ : the per unit cost for the chairs (RM175)

$R$ : the initial price per unit for the chairs (RM250)

$S$ : the number of units ordered (3000)

$V$ : the number of units sold during the first eight weeks the promotion (2000).

Solution
----------

$$P = (R - C)V + \left(\frac{R}{2} - C\right)(S - V)$$

$$P = ?$$

Model for special furniture sale	
Stock order_S	3000
Unit Cost_C	\$175.00
Demand_V	2000
Selling price(initial)	\$250.00
Revenue within 8 weeks	6000000
Profit	\$150,000.00
After 8 weeks:	
Amount sale(Quantity)	1000
Selling price (R/2)	\$125.00
Profit	-\$50,000.00
Total profit	\$100,000.00



## 2.5 Dynamic System Simulation on Spreadsheet

1. A dynamic system simulation model differs from a Monte Carlo simulation in two principal ways.
  - a) Involves the passage of time and an explicit representation of the sequence in which events occur.
  - b) It often describes the flow of some type of entity through the system.
2. Entity
  - a) A physical object, such as job being processed in a factory or inventory being transferred from a warehouse to a customer.
  - b) A piece of information, such as a message sent through communication system or a job waiting to process at a central computer center.
3. The goal of dynamic system simulation model is to reproduce the activities that control the flow of entities and the logic by which events occur over time.
4. Dynamic system simulation models can be formulated in several ways.
  - a) First approach, we could describe the activities that occur during fixed intervals of time, such as a week, day or hour, this is called activity scanning. (For example, to model the operation of an inventory system, we could describe the sequence of events that occur during a specific time period: fulfilling customer demand, ordering new stock and receiving stock that was ordered at an earlier time). Then we advance time to the next period and repeat.
  - b) Second approach, process-driven simulation describes the process through which entities in the system flow. (For example, in a service system, customers arrive, wait in line if the server busy, receive service and then leave the system. Process-driven simulation models the logical sequence of events for each customer as he or she arrives).
  - c) Third approach, event-driven simulation describe the changes that occur in the system at the instant of time that each event occurs. Events are sequenced in chronological order. (For example, in the service system, the key events are the arrival of customers, the start service and the end service. The arrival of the second customer might precede the starting time of service for the first customer. The simulation logic would describe what happens when customer 1 arrives, then when customer 2 arrives and then when customer 1 starts service).
5. Dynamic system simulation models are more complex than Monte Carlo models and difficult, if not impossible, to implement on spreadsheets. Certain activity scanning and process driven models are suitable for spreadsheet implementation; event driven models generally require more advanced software.

## 2.6 Basic concepts of Inventory Modeling

1. The two basic inventory decisions that manager face are how much to order or produce additional inventory and when to order or produce it.
2. The objective is to minimize total inventory costs.

### 2.6.1 Inventory costs

- a) Holding or carrying costs represent costs associated with maintaining inventory.
- b) Ordering costs represent costs associated with replenishing inventories. In manufacturing, setup cost is the equivalent to ordering cost.
- c) Shortage or stock-out costs are those costs that occur when demand exceeds available inventory in stock. A shortage may be handled as a back order, in which a customer waits until the item is available or as a lost sale.
- d) Purchasing costs are what firms pay for material or goods.

### 2.6.2 Economic Order Quantity (EOQ) Model

- The simplest and most elementary inventory model.
- The objective of the EOQ model is to determine the optimal order quantity that will minimize the inventory cost.
- The EOQ assumes:
  - i) The demand for inventory occurs at a constant rate and is known with certainty.
  - ii) Inventory replenishment is instantaneous (orders are received all at once) and occur only when the inventory level reaches zero.
  - iii) Lead time,  $L$  - the time between placement and receipt of an order is constant.
  - iv) A fixed order quantity,  $Q$ , is always ordered.
  - v) No shortages are incurred.
  - vi) The holding cost per unit and ordering cost per order are constant.



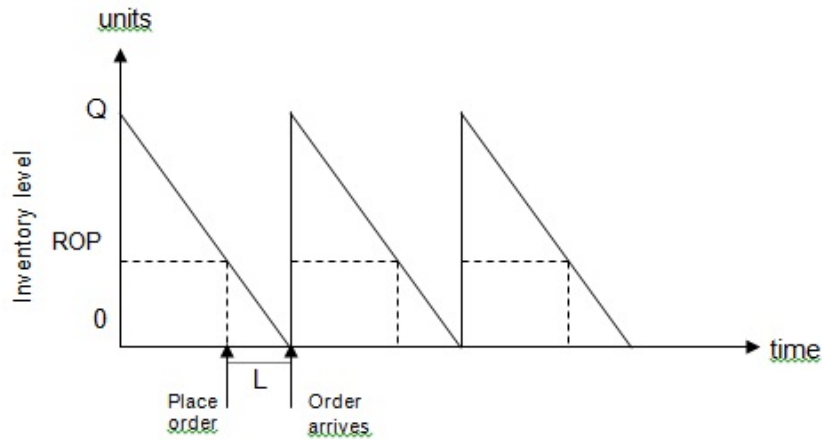


Figure 2.3: EOQ Inventory Process

### Economic Order Quantity (EOQ) Model

$$\begin{aligned}
 \text{Annual ordering cost} &= (\text{Number of order placed per year}) \times (\text{Ordering cost per order}) \\
 &= \frac{\text{Annual Demand}}{\text{Number of units in each order}} \times (\text{Ordering cost per order}) \\
 &= \frac{D}{Q} C_0 \quad (2.2)
 \end{aligned}$$

$$\begin{aligned}
 \text{Annual holding/carrying cost} &= (\text{Average inventory}) \times (\text{Carrying cost per unit per year}) \\
 &= \frac{\text{Order quantity}}{2} \times (\text{Carrying cost per unit per year}) \\
 &= \frac{D}{2} C_h \quad (2.3)
 \end{aligned}$$

$$\text{Optimal order quantity :} \quad (2.4)$$

$$EOQ, Q^* = \sqrt{\frac{2DC_0}{C_h}} \quad (2.5)$$

$$\begin{aligned}
 \text{Total annual cost : } TC &= \text{Order cost} + \text{Holding cost} \\
 &= \frac{D}{Q} C_0 + \frac{Q}{2} C_h \quad (2.6)
 \end{aligned}$$

$$\begin{aligned}
 ROP &= (\text{Demand per day}) \times (\text{Lead time for a new order in days}) \\
 &= d \times L \quad (2.7)
 \end{aligned}$$

When quantity discount are available, the purchase cost or material cos becomes are relevant cost. The total relevant cost are as follows:

$$\begin{aligned}
 \text{Total annual cost} &= \text{Material cost} + \text{Ordering cost} + \text{Carrying cost} \\
 &= DC + \frac{D}{Q} C_0 + \frac{Q}{2} C_h \quad (2.8)
 \end{aligned}$$

Where

- i)  $P$ =price per unit.
- ii)  $L$ =lead time (time between placed an order and receive the the order)
- iii)  $ROP$ =Reorder point, the inventory level at which an order should be placed.
- iv)  $C$ =cost per unit

**Example 2.6.1.**

We assume that demand is Poisson distributed with a mean of 100 units per week; thus the expected annual demand is 5200 units. It costs RM0.20 to hold 1 unit for 1 week ( $C_h = RM10.40$ ) and each order costs RM50. Every unfilled demand is lost and costs the firm RM100 in lost profit. The lead time between placing an order and the time the order is received is 2 weeks. Therefore, the expected demand during lead time is 200 units. Orders are placed at the end of the week and received at the beginning of the week.

**Solution** :

$$D = 5200 \text{ units}$$

$$C_0 = RM50$$

$$C_h = RM10.40$$

$$\begin{aligned} ROP &= 100 \text{ units per week} \times 2 \text{ weeks} \\ &= 200 \text{ units} \end{aligned}$$

The EOQ model suggests an order quantity,  $Q^* = \sqrt{\frac{2(5200)50}{10.40}} = 224 \text{ units}$

$$\text{Total annual cost : } TC = \frac{5200}{224}50 + \frac{224}{2}10.4 =$$

**Probabilistic Issues**

1. The EOQ model, as well as other classical analytical models, makes rather unrealistic assumptions about the constancy of demand rates and lead time.
2. In real life, demand and the lead time is usually uncertain.

### 2.6.3 System Simulation for Inventory Model

#### Example 2.6.2.

Saad, the owner of Saad Electrics wants to find a good, low cost inventory policy for one particular product: Ace Brand electric drill. He has decided to use simulation to determine this policy. Daily demand for the Ace model drill is relatively low but subject to some variability. Over the past 300 days, Saad has observed the sales of the drill and the data is given in the following table.

Demand for Ace drill	Frequency (days)
0	15
1	30
2	60
3	120
4	45
5	30
Total	300

When he places an order to replenish his inventory of the drills, there is a delivery lag of one to three days. The numbers of days it took to receive the past 50 orders is given in the following table.

Lead Time (days)	Frequency (orders)
1	10
2	25
3	15
Total	50

He wants to conduct a simulation analysis for an order quantity of 10 units with a reorder point of 5. That is, every time the on-hand inventory level at the end of the day is 5 or less. If the lead time is one day, it means that the order does not arrive the next day, but at the beginning of the following day. Initial inventory is 10 units.

Simulate this process for ten days.

From your simulation, determine:

- The average daily demand for Ace drill.
- The average lost sales.
- The number of orders placed.
- The probability that demand per day exceed 3 units.

Random Numbers:

Demand : 11 89 87 59 66 53 45 56 22 49

Lead Time: 29 74 80 73 56 77 60 65 87 18

## 2.7 Exercises

### Question 2.7.1.

Wira Plumbing Company maintains a stock of water filters that it sells to house owners and installs for them. Bahari, the sales manager of Wira Plumbing Company likes the idea of having large supply on hand to meet customer demand, but he also recognizes that it is expensive to do so. The sales for the water filters over the past 50 days has the following probability distribution:

Water filter sales per day	Probability
4	0.12
5	0.10
6	0.18
7	0.24
8	0.16
9	0.14
10	0.06

Lead-time can vary between zero and two days according to the following distribution.

Lead time (Days)	Probability
0	0.50
1	0.30
2	0.20

If the lead time is one day, it means that the order does not arrive the next day, but at the beginning of the following day. For each unit of demand that exceeds the stock, it will cost the company RM10 (i.e. stock out cost). For each water filter sold, it will yield a profit of RM15. The company orders 15 units of water filters when the ending inventory level reaches 5 units or less.

- a) Assuming the beginning inventory is 15 units, simulate the above problem for a 10-day period using the random numbers given below:

Random numbers for sales per day: 82 24 03 32 23 59 95 34 97 51

Random numbers for lead-time:

51 15 45 32 56 24 32 56 44 26

- b) Determine the average number of sales per day and the net profit for the 10-day simulation.

**Question 2.7.2.**

The distribution for weekly demand of Cobra auto alarm systems at Brother's Auto is as follows:

Demand	Probability
0	0.15
1	0.10
2	0.25
3	0.20
4	0.15
5	0.10
6	0.05

The alarm systems cost Brother's RM100 and sells for RM200. The weekly holding cost for these systems is 5%, and there is a RM65 cost to place an order. Lead-time can vary between one and three weeks according to the following probability distribution.

Lead time (weeks)	Probability
1	0.10
2	0.75
3	0.15

Assume that orders are placed at the end of the week. For example, if the lead-time is one week, an order placed in week 2 will arrive at the start of week 4. If the firm runs out of stock of the alarm systems, a customer will leave without making any purchase. Brother's Auto uses an inventory policy in which it orders 20 alarms whenever the stock on hand plus the units on order reaches 10 units. Assume the holding cost is calculated based on the end of the week inventory level.

- a) Using the random numbers given below, simulate the inventory for the Cobra alarm system at Brother's Auto over a 15-week period. Stock on hand at the beginning of the simulation exercise is 15 units.

Demand : 10 22 24 42 38 78 99 90 85 53 66 35 72 46 81  
 Lead time: 47 49 51

- b) Use your simulation to determine the total holding cost, total ordering cost, and total profit during the 15-week period.

**Question 2.7.3.**

A retailer sells an item for which the weekly demand can be described by the following distribution:

Demand	50	100	150	200	250	300	350	400	450
Probability	0.03	0.04	0.09	0.15	0.38	0.15	0.09	0.04	0.03

There is an initial inventory of 200 units of the item and the retailer orders in batches of 250 units whenever inventory level falls below 100. Past experience indicates that the lead time varies as follows:

Lead time (week)	1	2	3
Probability	0.25	0.50	0.25

Note: Orders are made at the end of the week and if the lead time is 1 week, the inventory is received after 1 full week.

The unit cost of holding is RM0.50 per week, applied to the total inventory held at each week ending. The cost of placing an order is RM50 and if there is no stock to satisfy a customer, the retailer suffers a loss of RM5 for each unit of unsatisfied demand.

- a) Determine the expected demand per week.
- b) Using the following random numbers, simulate the demand over a period of 15 weeks with the objective of obtaining the total inventory cost for each week.

Demand : 87 55 81 22 04 62 21 45 81 82 43 96 17 70 61

Lead time : 53 17 73 61 99

- c) From your simulation determine the followings:
  - (i) Demand per week
  - (ii) Total inventory cost

## 2.8 Basic concepts of Queuing Modeling

A waiting line (or queuing) system has three basic components: arrivals of entities to the system, waiting lines (or queues) and the service facility.

### i) Characteristics of a Queuing System

- Arrival
- Waiting line
- Service facility

### ii) Arrival Characteristics

#### 1. Size of the calling population

- Unlimited (infinite) Example: Cars arriving at a highway tollbooth, shoppers arriving at a supermarket or customers arriving at a bank.
- Limited (finite) Example: Shop with only receive eight breakdown machines at one time because of limited workers.

#### 2. Pattern of arrivals at the system

- Arrivals are considered random when they are independent of one another and their occurrence cannot be predicted exactly.
- Assumption for arrivals at service facility follows the Poisson distribution, for example 10 lorries per day, 20 customers per hour and 5 items per minute.

#### 3. Behavior of the arrivals

- Patient customer, wait in the queue until they are served.
- Balking customers who refuse to join the waiting line because it is too long to suit their needs or interests.
- Reneging customers are those who enter the queue but then become impatient and leave without completing their transaction.

### 2.8.1 Waiting line Characteristics

#### 1. The length of a line Unlimited Example: Case of tollbooth serving arriving automobiles. Limited Example: Case of small restaurant that has only 10 tables and can serve no more than 50 diners.

#### 2. Queue discipline that is the order in which customers are served

- First in first out Example: Payment counter at a supermarket.
- Priority Example: Hospital emergency room, patient who are critically injured will move ahead in treatment.
- Last in first out, random and prearranged schedule.

## 2.8.2 Service Facility Characteristics

1. The configuration of the service system
  - i) Number of channels number of servers
    - Single channel system one server
    - Multi channel system more than one server
  - ii) Number of phases number of stations
    - Single phase system is one in which the customer receives service from only one station and then exits the system.
    - Multi phase system implies two or more stops before leaving the system.
2. The pattern of service times
  - Assumptions for service times follows the Exponential distribution.

## 2.8.3 Basic Queuing System Configurations

### i) Single channel

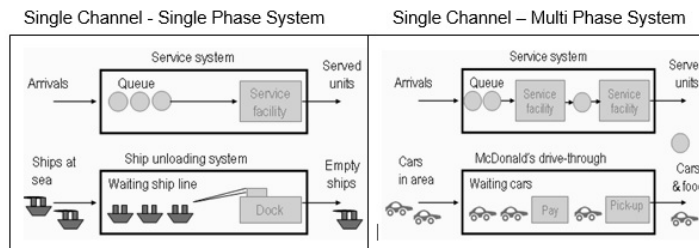


Figure 2.4: Single Channel

### ii) Multi Channel

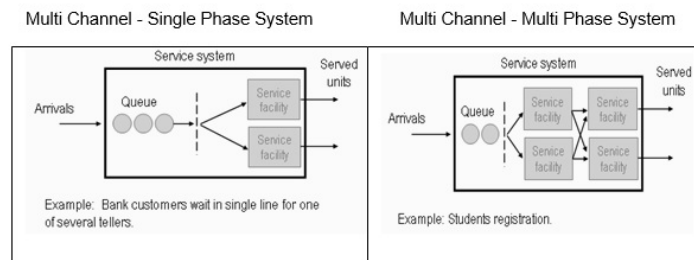


Figure 2.5: Multi Channel



### 2.8.4 Queuing Theory

- Is developed for predicting the characteristics of waiting line systems.

#### Single Channel Queuing Model

Assumptions of the model

1. An infinite calling population.
2. Arrival rate, follows Poisson distribution.
3. The queuing system is in normal / stable condition.
4. Unlimited queue length.
5. A first come first served queue discipline.
6. Service rate,  $\mu$ , follows Exponential distribution.

Average service rate  $>$  average arrival rate,  $\mu > \lambda$ , so that in real it is possible to serve all arrivals (customers are served at a faster rate than they arrive). If  $\lambda > \mu$ , then the queue will get larger and larger indefinitely.

The formulas for the operating characteristics for a single channel model

$\lambda$  = the average number of arrivals per time period

$\mu$  = the average number of customers or items served per time period

1. The average number of customers or units in the system (waiting in queue and being served).

$$L = \frac{\lambda}{\mu - \lambda} \quad (2.9)$$

2. The average number of customers in the queue.

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \quad (2.10)$$

3. The average time a customer spends in the system (total time in queue and counter).

$$W = \frac{1}{\mu - \lambda} \quad (2.11)$$

4. The average time a customer spends waiting in the queue.

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} \quad (2.12)$$

5. The utilization factor for the system, that is the probability that the service facility is being used (the server is busy).

$$\rho = \frac{\lambda}{\mu} \quad (2.13)$$

### 2.8.5 Multi Channel Queuing Model

Assumptions of the model

1. An infinite calling population.
2. Arrival rate,  $\lambda$ , follows Poisson distribution.
3. The queuing system is in normal / stable condition.
4. Identical counters / servers.
5. Unlimited queue length.
6. A first come first served queue discipline.
7. Service rate,  $\mu$ , follows Exponential distribution.

$s\mu > \lambda$ , the total number of servers must be able to serve customers faster than they arrive.

The formulas for the operating characteristics of a multi channel model

$\lambda$  = the average number of arrivals per time period

$\mu$  = the average number of people or items served per time period

$s$  = number of servers

1. The utilization factor.

$$\rho = \frac{\lambda}{s\mu} \quad (2.14)$$

2. The probability that there are zero customers or units in the system.

$$P_0 = \frac{1}{\left[ \left( \sum_{n=0}^{s-1} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n \right) + \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \left( \frac{1}{1-\rho} \right) \right]} \quad (2.15)$$

3. The average number of customers in the queue.

$$L_q = \frac{\rho \left( \frac{\lambda}{\mu} \right)^s P_0}{S!(1-\rho)^2} \quad (2.16)$$

4. The average number of customers or units in the system (waiting in queue and being served).

$$L = L_q + \frac{\lambda}{\mu} \quad (2.17)$$

5. The average time a customer spends in the system (total time in queue and counter).

$$W = \frac{L}{\lambda} \quad (2.18)$$

6. The average time a customer spends waiting in the queue.

$$W_q = \frac{L_q}{\lambda} \quad (2.19)$$

### Simulating Waiting Line System

These queuing theory formulas provide expected values for the operating characteristics but they provide no indication of the variability in performance. In a real system, we typically see large fluctuations around the averages.

Using simulation as a technique to study the operating characteristics of waiting line system, it will provide about the dynamic behavior of waiting lines.

#### Example 2.8.1. Queuing problem

The photocopy center at the Business Management Faculty has one copying machine. The interarrival time of students at the machine are as follows:

Interarrival time (minutes)	Probability
1	0.20
2	0.25
3	0.40
4	0.10
5	0.05

Photocopying is done on the first come first served basis. Analysis of past data has shown that the amount of time required for each student to make photocopies has the distribution shown below:

Photocopying time (minutes)	Probability
3	0.20
4	0.25
5	0.35
6	0.15
7	0.05

The owner feels that under the present situation, the waiting lines in front of the copying machine are too long and that the time a student spends queuing and waiting for the service is excessive. He wants to better serve the students.

- a) Assume the center opens at 9.00 a.m., perform a simulation for 15 arrivals using the following random numbers: Interarrival times : 76 90 04 65 44 98 60 16 84 34 69 98 24 59 23  
Photocopying times: 21 34 75 40 18 51 96 37 82 71 65 02 94 85 63
- b) Based on the simulation in a), answer the following questions.
- By 9.30 a.m. how many students have arrived? How many are served?
  - The owner will only provide another machine to better serve the students if the average time students spend queuing is more than 10 minutes. Should he add another machine

## 2.9 Exercises

### Question 2.9.1.

HiTec Computers is considering using e-commerce to sell its computers directly over the internet. It is planning to monitor the website 24 hours a day with a supervisor and a trainee. If an order arrives when both of them are free, the order will be processed by the supervisor. However, if both of them are busy, the order will be put in a queue until one of them is available. Operations data are estimated to be as follows:

Time Between Orders (minutes)	Probability	Supervisors Service Time (minutes)	Probability
2	0.35	4	0.30
3	0.30	5	0.25
4	0.2	6	0.25
		7	0.20

Trainees Service Time (minutes)	Probability
6	0.10
7	0.10
8	0.60
9	0.20

- a) Develop a simulation model for this problem. Run the simulation for 10 orders. The simulation should include the arrival of order time, start and end service times, and waiting time in the queue. Start your simulation at 8.00 am.
- b) The management of the company is interested in the average amount of time incoming orders will spend in the queue.

Use the following random numbers for time between orders and service time.

Time between orders: 06 61 70 88 11 52 82 12 35 99

Service time : 38 74 31 39 51 83 60 81 36 96

**Question 2.9.2.**

Penang International Airport primarily serves domestic air traffic. However, chartered planes from abroad may occasionally arrive with passengers bound for Langkawi Island and other tourist destinations around Penang. There is one immigration officer and one custom officer available at any time. Whenever an international plane arrives at the airport, the immigration and customs officers on duty will set up operations to process the passengers. Incoming passengers must first have their passports and visas checked by the immigration officer. The time required to check a passenger's passport and visa can be described by the following probability distribution:

Time required to check a passengers Passport & Visa (seconds)	Probability
20	0.20
40	0.40
60	0.10

After having their passports and visas checked, the passengers proceed to the customs officer who will inspect their baggage. Passengers form a single waiting line and baggage are inspected on a first come, first serve basis. The time required for baggage inspection has the following probability distribution:

Time required for baggage inspection (minutes)	Probability
0	0.25
1	0.60
2	0.10
3	0.05

- a) Suppose a chartered plane from abroad with 100 passengers lands at Penang Airport. Simulate the immigration and customs clearance process for the first 10 passengers and determine how long it will take them to clear the process.

Use the following random numbers for passport control: 93 63 26 16 21 26 70 55 72 89

and the following random numbers for baggage inspection: 13 08 60 13 68 40 40 27 23 64

- b) What is the average length of time a customer has to wait before having his baggage inspected after clearing passport control?

**Question 2.9.3.**

Com War Sdn. Bhd. produces monitors and printers for computers. Currently, all the monitors and printers are channeled to an inspection station, one at a time as they are completed. The interarrival time (in minutes) for the monitors has the following probability distribution:

Interarrival Time (minutes)	Probability
10	0.05
12	0.10
14	0.20
16	0.30
18	0.25
20	0.10

The interarrival time for the printers, on the other hand, is constant at 15 minutes. The inspection station has two inspectors. One inspector works on the monitors only and the other one inspects the printers only. In either case, the inspection time (in minutes) has the following probability distribution:

Inspection Time (minutes)	Probability
5	0.15
10	0.25
15	0.40
20	0.20

The management wants to evaluate the waiting time for monitors at the inspection station.

- a) Perform a simulation on the arrivals of the first 10 monitors at the inspection station. From the simulation, compute the average waiting time before beginning of inspection. Use the following random numbers:

Interarrival Time: 55 27 78 09 86 44 37 95 15 62

Inspection Time: 63 12 98 35 48 87 05 71 24 56

The management is considering training the two inspectors to work on either product. This is to prepare them for a new inspection procedure. Under the new procedure, a finished product will be channeled to either inspector for inspection.

- b) Consider the arrivals of the first 10 products (printers and monitors) at the inspection station. Perform a simulation to determine the average waiting time (before inspection) for the monitors and printers. Use the random numbers from part (a).

# Chapter 3

## Probability Distributions in Simulation

### 3.1 The important issues

1. Probability distributions are fundamental to simulation modeling. Real systems exhibit a great deal of random variation.
2. For example in a communication system, the time between message arrivals to the system and the length of time a message takes typically are random variables.
3. In manufacturing operations, times of job arrivals, job types, processing times, times between machine breakdowns, and repair times are random variables.
4. One important element of simulation modeling is identifying appropriate probability distributions for input data. Often, this requires analyzing empirical or historical data and fitting these data to a distribution.
5. A random variable assigns a numerical value for every outcome of the experiment. Random variable can be either a discrete or continuous. A discrete random variable assumes countable values. E.g. number of cars, number of houses, number of complaints. A continuous random variable can assume any value in one interval. E.g. height, time, amount of milk in a gallon, weight, price.

#### 3.1.1 How to represent random variable in the system with random numbers?

1. In simulation model for waiting line system, inter arrival times and service times are the two important inputs.
2. Let say, the data collection of 5 customers inter arrival times for payment counter system are as follow:

Customer	Inter arrival time (minutes)
1	1
2	1
3	2
4	3
5	2

- From the real data collection of inter arrival times, we need to identify the best probability distribution. Therefore, in the simulation model, the inter arrival times and service times probability distribution based on the real data.
- The probability distribution for the above data is given below:

Inter arrival time (minutes)	Probability distribution	Cumulative probability distribution	Random number interval
1	0.40	0.40	01 - 40
2	0.40	0.80	41 - 80
3	0.20	1.00	81 - 00

- Assume that random number generation can generate outcomes for example in simulation data set (1), (2) and (3).

Customer	Inter arrival time (1)	Inter arrival time (2)	Inter arrival time (3)
1	2	2	2
2	2	1	1
3	1	3	1
4	1	1	2
5	3	2	3

- Based on simulation data set (1), (2) and (3) the occurrence of inter arrival times data are different but the value of the probability distribution are the same.
- Therefore, random number generation must generate outcomes from any probability distribution, for example uniform distribution, exponential distribution, normal distribution and many others.

### 3.1.2 Random Variates

- Before this we saw how Excel can generate random numbers, uniformly distributed numbers between 0 and 1. We need for generating outcomes from other types of probability distributions.
- An outcome generated from a probability distribution is called a random variate. [A random number is actually a random variate from a uniform (0,1) distribution.]



3. The basic approach for generating random variate is to transform one or more random numbers into a random variate from an arbitrary uniform distribution with parameters  $a$  and  $b$ . Consider the formula

$$x = a + (b - a)R \quad (3.1)$$

where  $R$  is a random number. Note that when  $R = 0$ ,  $x = a$ , and when  $R = 1$ ,  $x = b$ . For any other value of  $R$  between 0 and 1,  $(b - a)R$  represents the same proportion of the interval  $(a, b)$  as  $R$  of the interval  $(0,1)$ . Then, all real numbers between  $a$  and  $b$  can occur. Since  $R$  is uniformly distributed, so also is  $x$ .

4. How does one transform a sample of the uniform  $[0,1]$  random variable into a sample of a given distribution? Method for transformation we can use is the inverse transformation method. Other methods we can use are convolution method, composition method and acceptance-rejection method.

### 3.1.3 The Inverse Transformation Method

Given  $f(x)$ , probability distribution function.

STEP 1: Obtain the cumulative probability distribution function  $F(x)$ , where

$$F(x) = \int_a^x f(x) dx, \quad a \leq x \leq b \quad (3.2)$$

STEP 2: Generate random number  $R$ , for  $0 < R < 1$ .

STEP 3: Set  $F(x) = R$  and solve for  $x$ . This value of  $x$  is the desired random observation from the probability distribution.

#### Example 3.1.1.

The Bigelow Manufacturing Company produces a product on a number of machines. The elapsed time between breakdowns of the machines is defined by the following continuous probability distribution

$$f(x) = \frac{x}{8}, \quad 0 \leq x \leq 4$$

where  $x$  = weeks between machine breakdown.

When a machine breaks down, it must be repaired and it takes either one, two or three days for the repair to be completed. The repair time follows a discrete probability distribution given in the following table:

Machine repair time (days)	Probability	Cumulative probability	Random number range
1	0.15	0.15	01-15
2	0.55	0.70	16-70
3	0.30	1.00	71-00

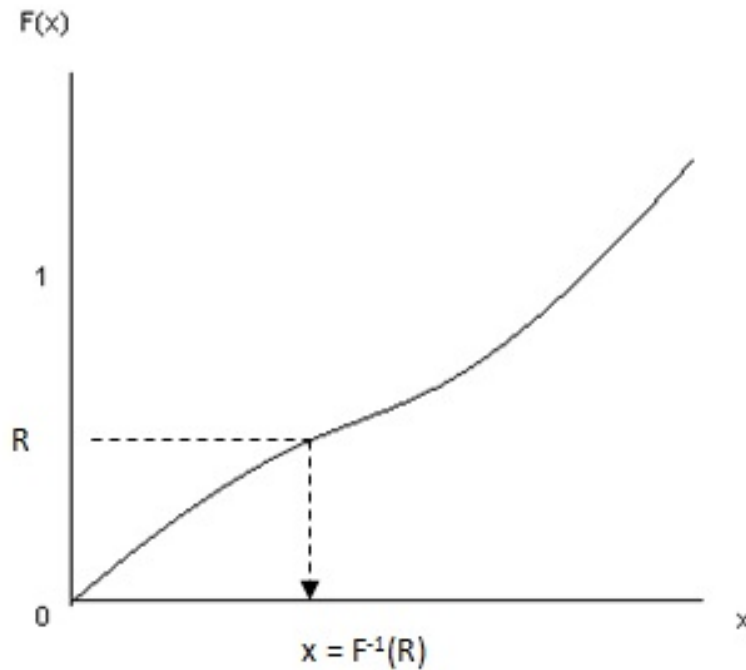


Figure 3.1: Graphical illustration of the inverse transformation method for obtaining a random observation from a given probability distribution

Every time a machine breaks down, the cost to the company is estimated RM2000 per day in lost production until the machine is repaired. Simulate this problem for 20 breakdowns. What is the total cost for 20 breakdowns?

Random numbers:

Time between breakdown

45 90 84 17 74 94 07 15 04 31 07 99 97 73 13 03 62 47 99 75

Repair time

19 65 51 17 63 85 37 89 76 71 34 11 27 10 59 87 08 08 89 42

**Solution** :

First we must find the cumulative probability function for the time between machine breakdowns. Consider the continuous probability function:

$$f(x) = \frac{x}{8}, \quad 0 \leq x \leq 4$$

- The area under the curve,  $f(x)$  represents the probability of the occurrence of the random variable  $x$ .
- Therefore, the area under the curve must equal 1.0 because the sum of all probabilities of the occurrence of a random variable must equal 1.0.
- By computing the area under the curve from 0 to any value of the random variable,  $x$ , we can determine the cumulative probability of that value of  $x$ , as follows:

$$\begin{aligned} F(x) &= \int_0^x \frac{x}{8} dx \\ &= \frac{x^2}{2(8)} \Big|_0^x \\ &= \frac{x^2}{16} \end{aligned}$$

- Cumulative probabilities are similar to the discrete ranges of random numbers we used in previous examples.
- Thus, we let this function,  $F(x)$ , equal the random number,  $R$ :

$$R = \frac{x^2}{16}$$

and solve for  $x$  gives

$$x = 4\sqrt{R}$$

- By generating random number,  $R$  and substituting it into this function, we determine a value for  $x$ , the time between breakdown.
- However, for a continuous function, the random numbers must be between zero and one to correspond to probabilities between 0.0 and 1.0.
- For example, if  $R = 0.25$ , then  $x = 4\sqrt{0.25} = 2$  minutes.
- So, to simulate this problem, we must obtain two random numbers, one for time between breakdowns and the other one machine repair time.

Breakdown	Rn1	Time between breakdown	Rn2	Repair time	Cost (RM000)
1	0.45	2.68	19	2	4
2	0.90	3.80	65	2	4
3	0.84	3.67	51	2	4
4	0.17	1.65	17	2	4
5	0.74	3.44	63	2	4
6	0.94	3.88	85	3	6
7	0.07	1.06	37	2	4
8	0.15	1.55	89	3	6
9	0.04	0.80	76	3	6
10	0.31	2.23	71	3	6
11	0.07	1.06	34	2	4
12	0.99	3.98	11	1	2
13	0.97	3.94	27	2	4
14	0.73	3.42	10	1	2
15	0.13	1.44	59	2	4
16	0.03	0.70	87	3	6
17	0.62	3.15	08	1	2
18	0.47	2.74	08	1	2
19	0.99	3.98	89	3	6
20	0.75	3.46	42	2	4
				Total cost	= RM84000

## 3.2 Continuous Distributions

### 3.2.1 Uniform Distributions

1. The uniform distribution characterizes a random variable  $X$  for which all outcomes between some minimum and maximum value are equally likely.
2. Let the random variable  $X$  denote the outcome when a point is selected at random from an interval  $[a, b]$ ,  $-\infty < a < b < \infty$ .
3. If the experiment is performed in a fair manner, it is reasonable to assume that the probability that the point is selected from the interval  $[a, b]$ ,  $a \leq x < b$  is  $\frac{(x-a)}{(b-a)}$  (Hogg, 2010).
4. Because  $X$  is a continuous -type random variable,  $F'(x)$  is equal to the probability function (P.d.f) of  $X$ .
5. For a uniform distribution with a minimum value  $a$  and a maximum value  $b$ , the

probability density function is

$$f(x) = \frac{1}{(b-a)}, \quad \text{if } a \leq x \leq b \quad (3.3)$$

The distribution function is

$$F(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ 0, & \text{if } x > b \end{cases} \quad (3.4)$$

$$\text{mean, } E(x) = \mu = \frac{a+b}{2}$$

$$\begin{aligned} \mu &= \int_a^b x f(x) dx \\ &= \frac{1}{2(b-a)} x^2 \Big|_a^b \\ &= \frac{1}{2(b-a)} (b^2 - a^2) \\ &= \frac{1}{2(b-a)} (b+a)(b-a) \\ &= \frac{1}{2} (a+b) \end{aligned}$$

$$\text{Variance, } \sigma^2 = \frac{(b-a)^2}{12}$$

$$\sigma^2 = E(X^2) - [E(x)]^2 \quad (3.5)$$

$$\begin{aligned} E(X^2) &= \int_a^b x^2 \frac{1}{(b-a)} dx \\ &= \frac{1}{3(b-a)} x^3 \Big|_a^b \\ &= \frac{1}{3(b-a)} (b^3 - a^3) \\ &= \frac{1}{3(b-a)} (b-a)(b^2 + ab + a^2) \\ &= \frac{1}{3} (b^2 + ab + a^2) \\ \sigma^2 &= \frac{1}{3} (b^2 + ab + a^2) - \left( \frac{a+b}{2} \right)^2 \\ &= \frac{1}{12} [4(b^2 + ab + a^2) - 3(a^2 + 2ab + a^2)] \\ &= \frac{b^2 - 2ab + a^2}{12} \\ &= \frac{(b-a)^2}{12} \end{aligned} \quad (3.6)$$

6. This distribution is also referred to as **rectangular**, because the graph of  $f(x)$  suggests the name.

7. Parameters  $a, b$

Excel formula to generate random variate,  $x = a + (b - a)R$

The uniform distribution is often used when little knowledge about a random variable; the parameter  $a$  and  $b$  are chosen judgmentally to reflect a modeler's best guess about the range of the random variable.



Figure 3.2: Uniform Distribution.

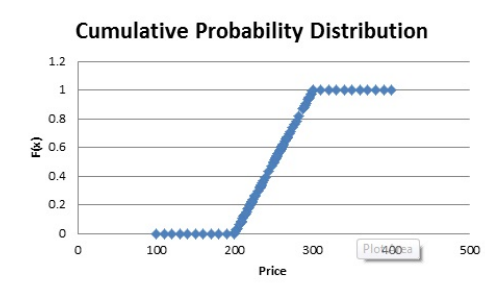


Figure 3.3: Cumulative probability distribution.

### 3.3 Exponential Distributions

The exponential distributions model events that recur randomly over time. Thus, it is often used to model the time between customer arrivals to service system and the time to failure of machines, light bulbs, and other mechanical or electrical components. A key property of the exponential distribution is that it is memory less; that is the current time has no effect on future outcomes. For example, the length of time until a machine failure has the same distribution no matter how long the machine has been running. It describes for example time between arrivals in a process which arrivals occur continuously and independently at a constant average rate. The exponential distribution has the density function

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0 \quad (3.7)$$

And distribution function

$$F(x) = 1 - e^{-\lambda x}, \quad x \geq 0 \quad (3.8)$$

Mean =  $\frac{1}{\lambda}$ , Variance =  $(\frac{1}{\lambda})^2$

Excel formula to generate random variate,  $x = -\frac{1}{\lambda} \ln(1 - R)$

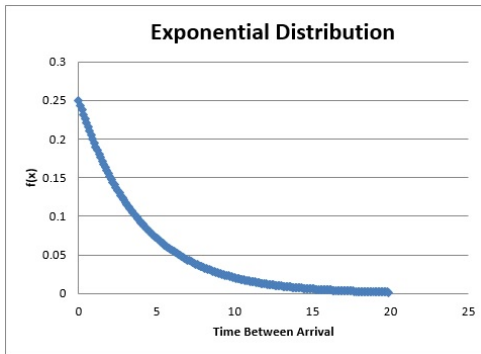


Figure 3.4: Exponential Distribution.

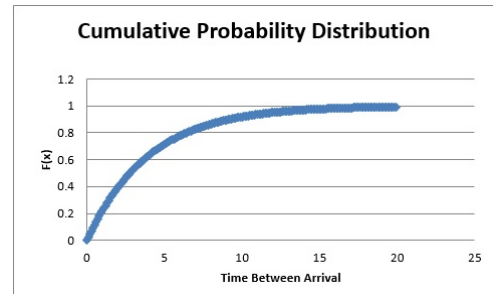


Figure 3.5: Cumulative probability distribution.

### 3.4 Normal Distributions

The normal distribution is described by the familiar bell-shaped curve. The normal distribution is symmetric and has a property that the median equals the mean. The normal distribution is observed in many natural phenomena. Processing times in some service systems follow a normal distribution. It is characterized by two parameters: the mean,  $\mu$  (location of the peak), and the variance,  $\sigma^2$  (the measure of the width of the distribution). The probability density function for the normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad \infty < x < \infty \quad (3.9)$$

Standard normal distribution, with  $\mu = 0$  and  $\sigma = 1$ . Parameter  $\mu$ ,  $\sigma$ . Excel formula to generate random variate,  $x = \text{NORMINV}(\text{RAND}(), \text{mean}, \text{standard deviation})$

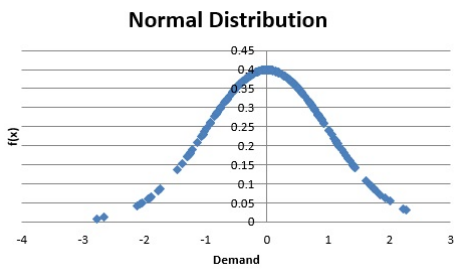


Figure 3.6: Normal Distribution.

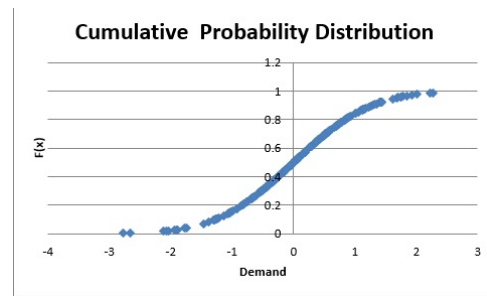


Figure 3.7: Cumulative probability distribution.



### 3.5 Triangular Distributions

The triangular distribution is defined by three parameters: the minimum,  $a$ , the maximum  $b$ , and the most likely,  $c$ . Outcomes near the most likely value have a higher chance of occurring than those at the extreme. By varying the position of the most likely value relative to the extremes, the triangular distribution can be symmetric or skewed in either direction. The probability density function is given by:

$$f(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)}, & \text{if } a \leq x \leq c \\ \frac{2(b-x)}{(b-a)(c-a)}, & \text{if } b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \quad (3.10)$$

The distribution function is

$$f(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{(x-a)^2}{(b-a)(c-a)}, & \text{if } a \leq x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(c-a)}, & \text{if } b \leq x \leq c \\ 1, & \text{if } x > b \end{cases} \quad (3.11)$$

The mean is computed as  $\frac{a+b+c}{3}$  and the variance is  $\frac{a^2+b^2+c^2ab-ac-bc}{18}$ .

Parameter:  $a, b, c$  Excel formula to generate random variate,  $x = a + \frac{b-a}{2}(R_1 + R_2)$

Symmetric Triangular

The triangular distribution is often used as a rough approximation of other distributions or if there is only limited sample data, because it depends on three simple parameters.

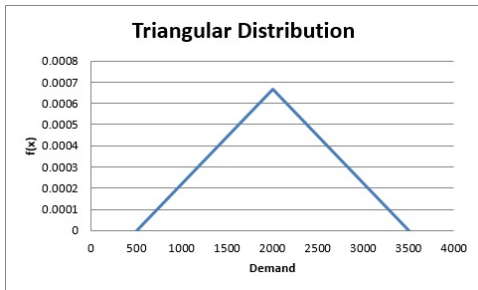


Figure 3.8: Triangular Distribution.

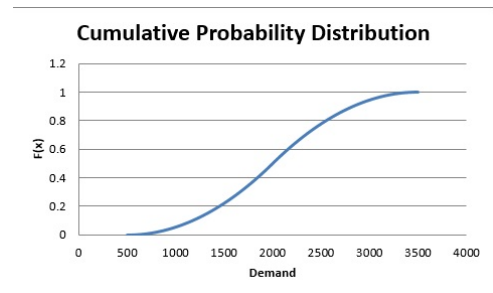


Figure 3.9: Cumulative probability distribution.

## 3.6 Lognormal Distributions

A lognormal distribution is a probability distribution of a random variable whose logarithm is normally distributed. The probability density function is

$$f(x) = \begin{cases} \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(\ln(x)-\mu)^2}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases} \quad (3.12)$$

The mean is  $e^{\mu+\frac{\sigma^2}{2}}$  and the variance is  $e^{2\mu+2\sigma^2}(e^{\sigma^2}-1)$ .  $\sigma$  must be greater than 0. It finds application in modeling phenomena that have low probabilities of large values and cannot have negative values, such as time to complete a task, stock prices and real estate prices. To generate a random number from a lognormal distribution with mean,  $\mu$  and standard deviation,  $\sigma$  you would use the following formula in Excel:

$$x = \text{LOGINV}(\text{RAND}(), \text{mean}, \text{standard deviation})$$

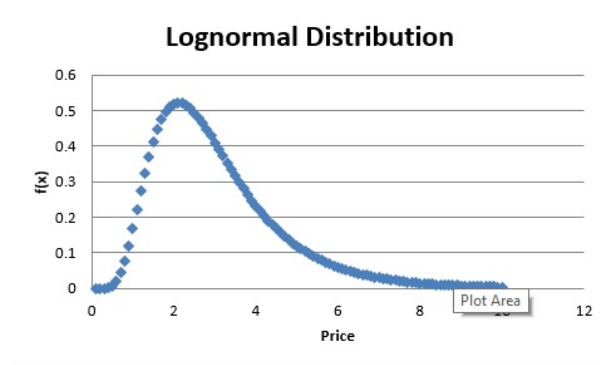


Figure 3.10: Lognormal Distribution.

## 3.7 Discrete Distributions

### 3.7.1 Bernoulli Distributions

The Bernoulli distribution characterizes a random variable with two possible outcomes with constant probabilities of occurrence. Typically these outcomes represent success ( $x = 1$ ) or failure ( $x = 0$ ). The probability mass function (p.m.f) is

$$p(x) = \theta^x(1 - \theta)^{1-x}, \quad x = 0, 1 \quad (3.13)$$

or

$$p(x) = \begin{cases} 1 - \theta, & \text{if } x = 0 \\ \theta, & \text{if } x = 1 \end{cases} \quad (3.14)$$

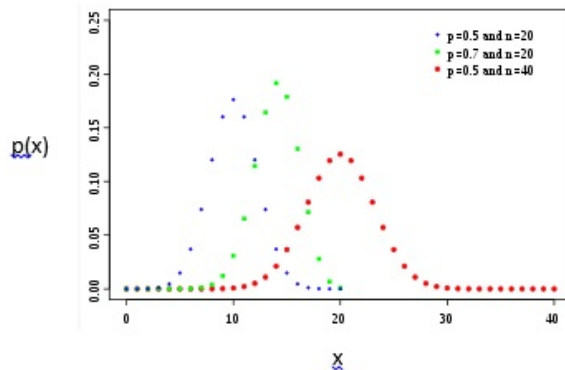
where  $\theta$  represents the probability of success,  $1 - \theta$  represents the probability of failure. A Bernoulli distribution might be used to model whether a person responds positively or not to a telemarketing promotion.

### 3.7.2 Binomial Distributions

The binomial distribution models  $n$  independent replications of a Bernoulli trial with probability  $p\theta$  of success on each trial. The random variable  $x$  represents the number of successes in these  $n$  trials. The probability mass function is

$$p(x) = \begin{cases} \binom{n}{x} \theta^x (1 - \theta)^{n-x}, & \text{if } x = 0, 1, \dots, n \\ 0, & \text{otherwise} \end{cases} \quad (3.15)$$

A binomial distribution might be used to model the results of the effects of drug research on a sample of patients.



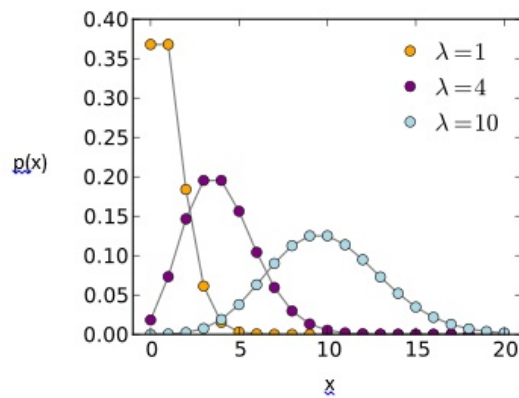
Source: [http://en.wikipedia.org/wiki/Binomial\\_distribution](http://en.wikipedia.org/wiki/Binomial_distribution)

Figure 3.11: Binomial Distribution.

### 3.7.3 Poisson Distributions

The Poisson distributions used to model the number of occurrences in some unit measure, for example the number of events occurring in an interval of time and the number of items demanded per customer. The Poisson distribution assumes no limit on the number of occurrences, that occurrences are independent and that the average number is constant. The probability mass function is

$$p(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & \text{if } x = 0, 1, 2, \dots, \infty \\ 0, & \text{otherwise} \end{cases} \quad (3.16)$$



Source: [http://en.wikipedia.org/wiki/Poisson\\_distribution](http://en.wikipedia.org/wiki/Poisson_distribution)

Figure 3.12: Poisson Distribution.

### 3.7.4 Other Useful Distributions

#### Gamma Distribution

1. Gamma distributions are often used to model the time to complete a task, such as customer service or machine repair.
2. A special case of Gamma distribution is called Erlang distributions.
3. Gamma distribution is a continuous probability distribution. Its Probability distribution function is

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-\frac{x}{\theta}}, \quad 0 \leq x < \infty \quad (3.17)$$

where  $\Gamma(\alpha) = (\alpha - 1)!$

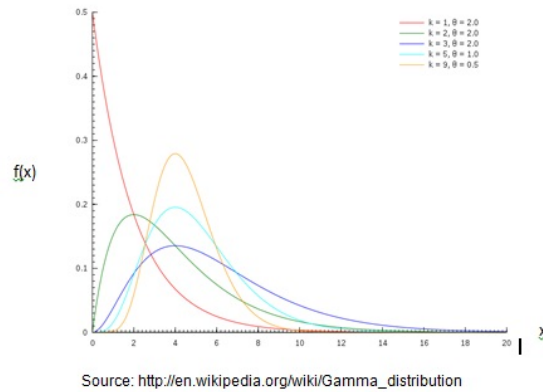


Figure 3.13: Erlang Distribution.

4. To generate a random number from a (2-parameter) Gamma distribution with  $\alpha$ , and  $\beta$ , you would use the following formula in Excel:  $x = \text{GAMMAINV}(\text{RAND}(), \alpha, \beta)$
5. Mean  $\mu = \alpha\theta$ , variance  $\sigma^2 = \alpha\theta^2$

### Weibull Distribution

1. Refer to page 170 Hogg (2010), Suppose that we now let the continuous-type random variable  $W$  be the interval necessary to produce the first change. The distribution function of  $W$  is

$$G(w) = P(W \leq w) = 1 - p(W > w), \quad 0 \leq w \quad (3.18)$$

2. The probability distribution function of  $W$  is

$$g(w) = G'(w) = H'(w)e^{-H(w)} = \lambda(w)e^{-\int_0^w \lambda(t) dt} \quad (3.19)$$

3.  $\lambda(w)$ , which commonly called the **failure rate** or **force or mortality**, is increasing function of  $w$  is given by

$$\lambda(w) = \frac{g(w)}{1 - G(w)} \quad (3.20)$$

**Example 3.7.1.**

Let

$$H(w) = \left(\frac{w}{\beta}\right)^\alpha, \quad 0 \leq w$$

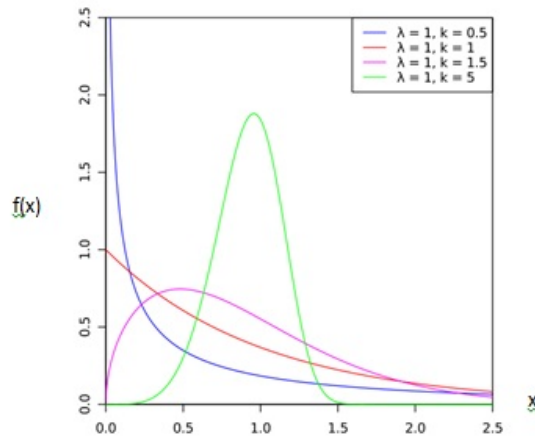
so that the failure rate is

$$\lambda(w) = H'(w) = \frac{\alpha w^{\alpha-1}}{\beta^\alpha}$$

where  $\alpha > 0$ ,  $\beta > 0$ . Then the pdf of  $W$  is

$$g(w) = \frac{\alpha w^{\alpha-1}}{\beta^\alpha} e^{-\left(\frac{w}{\beta}\right)^\alpha} \quad (3.21)$$

4. pdf (3.21) is called the **Weibull distribution**.
5. In engineering, this distribution, with appropriate  $\alpha$  and  $\beta$ , is excellent for describing the **life of a manufacturing item**.
6. The Weibull distribution is another probability distribution capable of taking on a number of different shapes. Both  $\beta$  and  $\beta$  must be greater than 0 and often used to model results from life and fatigue tests, equipment failure times and times to complete a task.



Source: [http://en.wikipedia.org/wiki/Weibull\\_distribution](http://en.wikipedia.org/wiki/Weibull_distribution)

Figure 3.14: Weibull Distribution.

7. To generate a random number from a (2-parameter) Weibull distribution with  $\alpha$ , and  $\beta$ , you would use the following formula in
8. Excel:  $x = \alpha[-\ln(1 - R)]^{\frac{1}{\beta}}$

### Beta Distributions

1. One of the most flexible distribution for modeling variation over a fixed interval from 0 to a positive value.
2. We say that  $Y$  has a **beta** pdf with parameter  $\alpha$  and  $\beta$  if (Hogg, 2010)(page 228)

$$g(y) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1} (1 - y)^{\beta-1}, \quad 0 < y < 1 \quad (3.22)$$

3. To generate a random number from a (2-parameter) beta distribution with  $\alpha$ , and  $\beta$ , you would use the following formula in Excel:  $x = \text{BETAINV}(\text{RAND}(), \alpha, \text{beta}, A, B)$

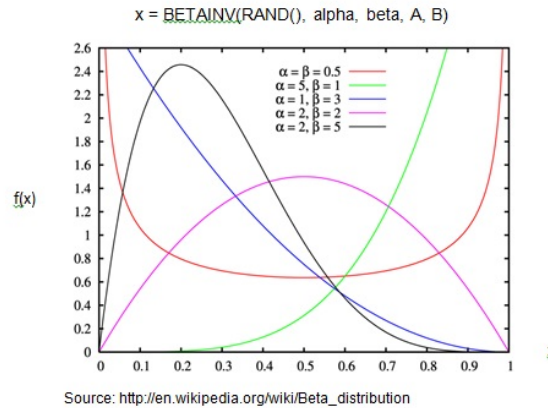


Figure 3.15: Beta Distribution.

## 3.8 Modeling probabilistic Inputs

There are wide varieties of probability distributions that we have just described, but it is not always easy to select the most appropriate.

## 3.9 Distribution Fitting of Empirical Data

For many inputs to simulation models, empirical data may be available, either in historical records or collected through special efforts.

### 3.9.1 Sale Simulation

**Example 3.9.1.** *Let us look at an example to understand the concept behind Monte carlo Simulation (Sambasivan and Yahya, 2002).*

PortaCom manufactures personal computer and related equipment. PortaCom's product design has developed a prototype for new high quality portable printer. The new printer has an innovative design and the potential to capture a significant share of the portable printer market. preliminary marketing and financial analyses have provided the following inputs:

Administrative cost-RM400,000

Advertising Cost-RM600,000

Selling price-RM249 per unit

Direct labor Cost; PortaCom believes that the direct labor cost will range from RM43 to RM47 per unit and is described by the discrete probability distribution as shown:

Direct Labor per unit (RM)	Probability
43	0.1
44	0.2
45	0.4
46	0.2
47	0.1

Parts Cost: This cost depends upon the general economy, the overall demand for parts, and the pricing policy of PortaCom's parts supplier. PortaCom believes that parts cost will range from RM80 to RM100 per unit and is described by a uniform probability distribution. Costs between RM80 and RM100 are equally likely. First-year demand: PortaCom be-

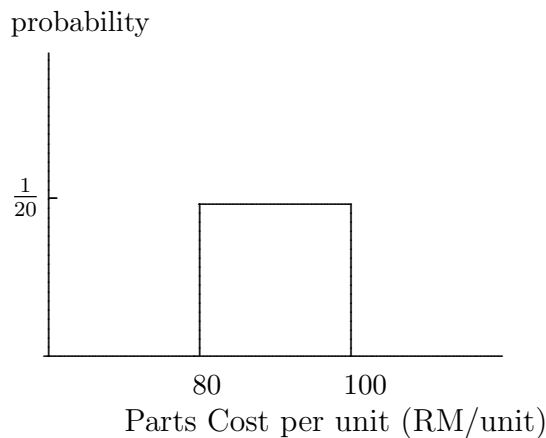


Figure 3.16: Uniform distribution

lieves that the first-year demand can be described by the normal probability distribution.



The mean or the expected value of the first-year demand is 15,000 units. The standard deviation of 4500 units described the variability in the first-year demand.

Simulate the problem for 10 trials and compute the average profit.

Let  $c_1$  be the direct labor cost per unit,  $c_2$  be the parts cost per unit, and  $x$  be the demand.  
profit= $(249 - c_1 - c_2)x - 1,000,000$

### Solution

1. In the simulation, representative values must be generated for the direct labor cost, parts cost, and the demand.
2. Random numbers and the probability distribution associated with each probabilistic input are used to generate representative values.
3. The computer generated random numbers are randomly selected decimal numbers from 0 up to, but not including 1.
4. The computer generated random numbers are equally likely are uniformly distributed between 0 and 1.
5. Computer generated random numbers can be obtained using built-in functions available in computer simulation packages and spreadsheets.
6. For instance placing =RAND() in a cell of an Excel spreadsheet will result in a random number between 0 and 1 being place in that cell.
7. Below is a list of 10 random numbers generated using Excel spreadsheet.
8. These random numbers are used to generate the values for direct labor cost, parts cost, and the first-year demand.
9. Calculation of parts cost: Let  $r$  be the random numbner between 0 and 1,  $a$  be the smallest value of the parts cost, and  $b$  the largest value of the parts cost:  
Parts Cost= $a + r(b - a)$

♣ Computer generated random numbers using EXCEL spreadsheet: Note:

Random Variable	
0.2568	0.8984
0.7049	0.9069
0.9739	0.4678
0.5303	0.6375
0.5641	0.3187
0.7197	0.7785
0.6518	0.9738
0.5889	0.4497
0.3542	0.3461
0.9926	0.8262

Figure 3.17: Random Numbers using EXCEL.

- To change random number: Highlight cell → click f9.
- To make the random number generated doesn't to change: Highlight → copy → paste.

♣ Random number intervals for generating values of direct labor cost per unit:

	A	B	C	D	E	F	G	H
1								
2		Direct labor cost per unit (RM)	Probability	Cumulative Probability	Interval of random numbers			Direct Labor Cost
3		43	0.1	0.1	0.0 but less than 0.1	0.0	0.1	43
4		44	0.2	0.3	0.1 but less than 0.3	0.1	0.3	44
5		45	0.4	0.7	0.3 but less than 0.7	0.3	0.7	45
6		46	0.2	0.9	0.7 but less than 0.9	0.7	0.9	46
7		47	0.1	1.0	0.9 but less than 1.0	0.9	1.0	47
8								

Figure 3.18: Random numbers interval of Labor cost.

	Key Cell Formulas	
Cell	Formula	Copied to
D3	=C3	High C3 → click D3
D4	=D3+C4	D5:D7

♣ Random generation of direct labor cost per unit:

Trial #	Random Number	Direct Labor Cost
1	0.26	44
2	0.70	46
3	0.97	47
4	0.53	45
5	0.56	45
6	0.72	46
7	0.65	45
8	0.59	45
9	0.35	45
10	0.99	47

Figure 3.19: Random numbers of Labor cost.

Key Cell Formulas		
Cell	Formula	Copied to
D10	=VLOOKUP(\$C10,\$F\$3:\$H\$7,3)	High D11 → click D19

♣ Random generation for the parts cost per unit

Part scost:		Random number(r')	part cost 80+r(100-80)	Random Variable	
Trial #					
1		0.8984	97.97	0.2568	0.8984
2		0.9069	98.14	0.7049	0.9069
3		0.4678	89.36	0.9739	0.4678
4		0.6375	92.75	0.5303	0.6375
5		0.3187	86.37	0.5641	0.3187
6		0.7785	95.57	0.7197	0.7785
7		0.9738	99.48	0.6518	0.9738
8		0.4497	88.99	0.5889	0.4497
9		0.3461	86.92	0.3542	0.3461
10		0.8262	96.52	0.9926	0.8262

Figure 3.20: Random numbers of Labor cost.

Key Cell Formulas		
Cell	Formula	Copied to
C22	=G22	Copy C23 → C31
D22	=80+C22*(100-80)	Copy C23 → C31

	Key Cell Formulas		
Title	Cell	Formula	Copied to
Direc labor	G36	=D10	Copy G37 → G45
parts Cost	H36	=D22	Copy H37 → H45
Units sold	I36	=D36	Copy I37 → I45
Profit	J36	=(249-G36-H36)*I36-1,000,00	Copy J37 → J45
Total	G46	=SUM(G35:G45)	Copy H46 → J46
Average	G47	=Average(G36:H45)	Copy H47 → J47
Std Devition	J48	=STDEV	

Table 3.1: Explanation of commands of spreadsheet

♣ Random generation for the parts cost per unit

Demand/unit sold
16860.35
11418.86
24234.40
18186.11
12662.07
11095.27
11469.95
15535.20
14090.63
21152.35

Figure 3.21: Random numbers of demand for 10 values.

♣ Portcom simulation results for 10 trials

$$\text{Profit} = (249 - c_1 - c_2)x - 1,000,000$$

Results:				
Trial	Direct Labor Cost (c1)	Parts Cost(c2)	Units Sold	Profit (RM)
1	44	97.97	16860.35	804586.96
2	46	98.14	11418.86	197409.43
3	47	89.36	24234.40	1729860.4
4	45	92.75	18186.11	1023195.3
5	45	86.37	12662.07	489391.76
6	46	95.57	11095.27	191970.87
7	45	99.48	11469.95	198890.69
8	45	88.99	15535.20	786628.19
9	45	86.92	14090.63	649712.39
10	47	96.52	21152.35	1231078.3
<b>Total</b>	<b>455</b>	<b>932</b>	<b>156705</b>	<b>7302724.26</b>
<b>Average</b>	<b>45.5</b>	<b>93.2</b>	<b>15670.5</b>	<b>730272.43</b>
<b>Std deviation</b>				<b>1950814.3</b>

Figure 3.22: Random numbers of demand for 10 values.

**Alternative Method:**

	A	B	C	D	E	F
49	Alternative(Direct )					
50		Trial	Direct labor Cost (RM)	Part cost (RM)	Units sold(Unit)	Profit (RM)
51		1	43	91.48	12298.6	408425.06
52		2	45	84.95	12487.0	486558.56
53		3	45	83.58	6911.8	-167657.05
54		4	46	83.06	13444.4	612505.83
55		5	43	98.29	6394.3	-311244.36
56		6	45	80.11	7282.3	-97790.41
57		7	44	82.50	14805.5	813624.73
58		8	47	95.50	11233.3	196384.79
59		9	46	99.36	16551.5	715373.04
60		10	45	86.40	12952.0	523111.09
61		Total	449	885.23	114360.7	3179291.29
62		Average	44.9	88.52	11436.1	317929.13
63		STd Devition				899168.09

Figure 3.23: Postacom simulation result.

	Key Cell Formulas		
Title	Cell	Formula	Copied to
Direc labor	C51	=Vlookup(RAND(),\$F\$3:\$7,3)	Copy C52→ C60
parts Cost	D51	=RAND()*(100-80)+80	Copy D52→D60
Units sold	E51	=NORMINV(RAND(),15000,4500)	Copy E52 → E60
Profit	F51	=(249-C51-D51)*E51-1,000,00	Copy F52 → F60
Total	C61	=SUM(C51:C60)	Copy D51 → F51
Average	C62	=Average(C51:C60)	Copypyp D62→ F62
STD Deviation	F63	=STDEV(F51:F60)	

Table 3.2: Explanation of command of alternative method

### 3.9.2 Inventory Simulation

**Example 3.9.2.** Refer to (Sambasivan and Yahya, 2002)

Butler Electrical Supply Company sells home ventilation fan. Each fan costs RM75 and butler sell them at RM125 making a gross profit of RM50 for each fan sold. Monthly demand for the fan is described by a **normal probability distribution** with a mean of 100 units and a standard deviation of 20 units. Butler receives monthly deliveries and replenishes its inventory to a level  $Q$  at the beginning of each month. If monthly demand is less than the replenishment level, an inventory holding cost of RM15 is charged for each unit that is not sold. However, if monthly demand is greater than the replenishment level, stock out occurs and a shortage cost is incurred. Because Butler assigns a goodwill cost of RM30 for each customer turned away, a shortage cost of RM30 is charged for each unit of demand that cannot be satisfied.

Management would like to use simulation to determine average net monthly profit resulting from using a particular replenishment level. The management would also like to know the percentage of demand satisfied for each replenishment level-service level.

Solution

*Case 1:* Demand less than or equal to replenishment level.

Gross profit=RM  $50D$ , where  $D$  is the demand

Holding cost=RM $15(Q - D)$ , where  $Q$  is the replenishment level

Net profit= $50D - 15Q + 14D = 65 - 15D$  (Gross profit-Holding cost)

*Case 2:* Demand more than the replenishment level

Gross profit=RM $50Q$ , where  $Q$  is the replenishment level

Shortage cost= RM $30(D - Q)$

Net profit= $50Q - 30D + 30Q = 80Q - 30D$  (Gross profit-Shortage cost)

In order to generate demand, use the Function NORMINV(RAND(),100,20) in Excel:95 108, 103 120, 129. Let us simulate the results for 5 trials:  $Q=100$ .

	A	B	C	D	E	F	G	H
1			Butler					
2		Q=100						
3		Month	Demand (D)	Sales	Gross Profit	Holding Cost	Shortage Cost	Net Profit
4		1	95	95	4750	75	0	\$ 4,675.00
5		2	108	100	5000	0	240	\$ 4,760.00
6		3	103	100	5000	0	90	\$ 4,910.00
7		4	120	100	5000	0	600	\$ 4,400.00
8		5	129	100	5000	0	870	\$ 4,130.00
9		Total	555	495	24750	75	1800	\$22,875.00
10		Average	111	99	4950	15	360	\$ 4,575.00
11								
12		Monthly Profit=	\$4,575.00					
13		Service level=	89.2%					

Figure 3.24: Butler simulation result.

	Key Cell Formulas		
Title	Cell	Formula	Copied to
Gross Profit	E4	=IF(C4 <= D4, 50 * C4, 50 * D4)	Copy E5 → E8
Holding Cost	F4	=IF(C4;=D4,15*(100-D4),0)	Copy F5 → F8
Shortage cost	G4	=IF(C4;D4,30*(C4-100),0)	Copy G5 → G8
Net Profit	G4	=IF(C4;=D4,E4-F4,E4-G4)	Copy G5 → G8
Total	C9	=SUM(C4:C8)	Copy D9 → G9
Average	C10	=Average(C4:C8)	Copy D10 → H10
Monthly net profit	H12	=H10	
Service level	C13	=D9/C9	click \$

Table 3.3: Explanation of command of Butler Simulation

**Note:**

1. The simulation runs can be done with  $Q = 110, 120, 130, 140$ . Service level can be calculated for each case, and the replenishment level,  $Q$ , corresponding to the acceptable service level can be chosen as as the **optimal level of  $Q$** .
2. Using EXCEL this problem can be run for 500 months and the average computed.
3. It can be seen that simulation is very to use.
4. In many situations, mathematical models for complex problems are validated using simulation.

# Chapter 4

## Simulation Examples

1. We will present several examples of simulations that can be performed by devising a simulation table either manually or with a spreadsheet.
2. This will provide insight into the methodology of discrete system simulation and the descriptive statistics used for predicting system performance.

### 4.1 Outline

1. The simulations are carried out by the following steps:
  - Determine the input characteristics.
  - Construct a simulation table.
  - For each repetition  $i$ , generate a value for each input, evaluate the function, and calculate the value of the response  $y_i$ .
2. Simulation examples will be given in queuing, inventory, reliability and network analysis.

### 4.2 Simulation of Queuing Systems

1. A queuing system is described by its calling population, nature of arrivals, service mechanism, system capacity and the queuing discipline.
2. In a single-channel queue:
  - The calling population is infinite.
  - Arrivals for service occur one at a time in a random fashion. Once they join the waiting line they are eventually served.
3. Arrivals and services are defined by the distribution of the time between arrivals and service times.



## 4. Key concepts:

- The system state is the number of units in the system and the status of the server (busy or idle).
- An event is a set of circumstances that causes an instantaneous change in the system state, e.g., arrival and departure events.
- The simulation clock is used to track simulated time.

## 5. Event list: to help determine what happens next:

- Tracks the future times at which different types of events occur.
- Events usually occur at random times.

## 6. The randomness needed to imitate real life is made possible through the use of random (pseudo-random) numbers (more on this later).

**4.2.1 Single-channel queue illustration**

- Assume that the times between arrivals were generated by rolling a die 5 times and recording the up face, then input generated is:

Customer	Interarrival Time	Arrival Time On Clock
1	-	0
2	2	2
3	4	6
4	1	7
5	2	9
6	6	15

- The first customer is assumed to arrive at clock time 0. The second customer arrives two time units later (at clock time 2), and so on.
- Assume the only possible service times are 1, 2, 3 and 4 time units and they are equally likely to occur, with input generated as:

Customer	Service time	Customer	Service time
1	2	4	2
2	1	5	1
3	3	6	4

- Resulting simulation table emphasizing clock times:

Customer Number	Time arrival (Clock)	Service time Begins(Clock)	Time Service (Duration)	End(Clock)
1	0	0	2	
2	2	2	1	3
3	6	6	3	9
4	7	9	2	11
5	9	11	1	12
6	15	15	4	19

- Another presentation method, by chronological ordering of events:

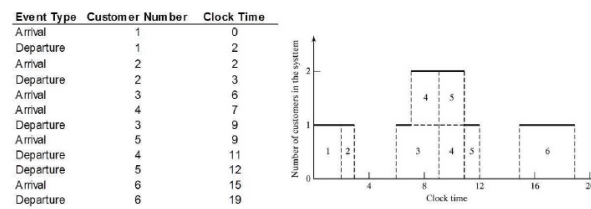


Figure 4.1: Butler simulation result.

## Grocery Store

- Grocery store example with only one checkout counter:

-Customers arrive at random times from 1 to 8 minutes apart, with equal probability of occurrence:

Time between Arrival(minutes)	Probability	Cumulative Probability	Random Digit Assignment
1	0.125	0.125	001-125
2	0.125	0.250	126-250
3	0.125	0.375	251-375
4	0.125	0.500	376-500
5	0.125	0.625	501-625
6	0.125	0.750	626-750
7	0.125	0.875	751-875
8	0.125	1.000	876-000

- The service times vary from 1 to 6 minutes, with probabilities:

Time between Arrival(minutes)	Probability	Cumulative Probability	Random Digit Assignment
1	0.10	0.10	01-10
2	0.2	0.30	11-30
3	0.30	0.60	30-60
4	0.25	0.85	61-85
5	0.10	0.95	86-95
6	0.05	1.00	96-00

- To analyse the system by simulating arrival and service of 100 customers:
  - Note this is chosen for illustration purposes, in actuality 100 customers is too small a sample size to draw any reliable conclusions.
  - Initial conditions are overlooked to keep calculations simple.
- A set of uniformly distributed random numbers is needed to generate the arrivals at the checkout counter:
  - Should be uniformly distributed between 0 and 1.
  - Successive random numbers are independent.
- With tabular simulations, random digits can be converted to random numbers:
  - List 99 random numbers to generate the times between arrivals.
  - Good practice to start at a random position in the random digit table and proceed in a systematic direction (never re-use the same stream of digits in a given problem).
- Generated time-between-arrivals

Customer	Random digits	Interarrival time (minutes)
1	-	-
2	64	1
3	112	1
4	678	6
5	289	3
6	871	7
⋮	⋮	⋮
100	538	4

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