UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering

CME263H1– QUIZ #3
Discrete Random Variables & Probability Distributions

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2018/02/15

Name: ____
Student Number: ____________________________________

This exam contains 7 pages (including this cover page) and 4 questions. Total of points is 30.
Good luck and Happy reading work!

Distribution of Marks

Question	Points	Score
1	8	
2	8	
3	6	
4	8	
Total:	30	

1. A contractor is required by a county planning department to submit one, two, three, four, or five forms (depending on the nature of the project) in applying for a building permit. Let $Y =$ the number of forms required of the next applicant. The probability that y forms are required is known to be proportional to y—that is, $p(y) = ky$ for $y = 1, \ldots, 5$.

   (a) (2 points) What is the value of k? (*Hint*: $\sum_{y=1}^{5} p(y) = 1$)

   (b) (2 points) What is the probability that at most three forms are required?

   (c) (2 points) What is the probability that between two and four forms (inclusive)are required?

   (d) (2 points) Could $p(y) = y^2/50$ for $y = 1, \ldots, 5$ be the *pmf* of Y.

2. The *pmf* of the amount of memory X (GB) in a purchased flash drive was given as

x	1	2	4	8	16
$p(x)$	0.05	0.10	0.35	0.40	0.10

Compute the following:

(a) (2 points) Expected value $E(X)$

(b) (2 points) Variance $V(X)$ directly from the definition

(c) (2 points) The standard deviation $\sigma(X)$

(d) (2 points) $V(X)$ using the shortcut formula ($V(X) = E(X^2) - E^2(X)$)

3. Each of 12 refrigerators of a certain type has been returned to a distributor because of the presence of a high-pitched oscillating noise when the refrigerator is running. Suppose that 5 of these 12 have defective compressors and the other 7 have less serious problems. If they are examined in random order, let X = the number among the first 6 examined that have a defective compressor. Compute the following:

   (a) (3 points) $P(X = 1)$

   (b) (3 points) $P(X \geq 4)$

4. A reservation service employs five information operators who receive requests for information independently of one another, each according to a Poisson process with rate $\mu = 2$ per minute.

   (a) (4 points) What is the probability that during a given 1-min period, the first operator receives no requests?

   (b) (4 points) What is the probability that during a given 1-min period, exactly four of the five operators receive no requests?(*Hint*: treat either as a binomial process of 5 trials with 4 successes or consider 5 combinations of Poisson processes, e.g. only 1st operation receives a request or only 2nd operation receives a request and so on)

Probability mass/distribution functions

Binomial Distribution

$$f(x;n,p) = b(x;np) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\mu = E(x) = np$$

$$\sigma_x^2 = np(1-p)$$

Hypergeometric Distribution

$$P(X = x) = h(x;n,M,N) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

$$\mu = E(X) = \frac{nM}{N}$$

$$\sigma_x^2 = n\frac{M}{N}\frac{N-M}{N}\frac{N-n}{N-1}$$

Poisson Distribution

$$P(x;\mu) = e^{-\mu}\frac{\mu^x}{x!}$$

$$E(X) = Var(X) = \mu$$

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.